1.4 Rounding

- Rounding to the nearest integer, or nearest 10, 100, etc. and decimal places (another way of saying "nearest tenth", "nearest hundredth", etc.) need to be clear before venturing into significant figures.
- It's important to think about how accurately numbers are needed in different contexts; e.g., football crowds to the nearest 1000, country populations to the nearest 1 000 000.
- The FIX mode on calculators can be helpful or unhelpful. The simplest way to turn it off is usually to use a biro to press the "reset" button at the back of the calculator.
- The *most significant digit* (the 1st significant figure) is the one that's worth the most, and it's the first non-zero digit from the left. The 2nd and subsequent significant figures *can* be zeroes (even though they're worth nothing); they are just the next digits you come to going to the right.

For example, in 4371, 4 is the most significant digit (put a "1" above it and write "2, 3, 4" above the next three digits – their "significance level"). To round 4371 to 1 sf means rounding to the nearest 1000 (the column that the 1st sf is in). Rounding 0.372 to 2 sf means rounding to the nearest hundredth (or 2 dp). Pupils sometimes confuse the most significant digit with the rounded answer. (4371 to 1 sf isn't just 4!) We need a number close to the number we're rounding – that's the whole point! On a number-line it's clearer that we're looking for a *close* number.

- "5 or more we round up, otherwise it stays the same" is a helpful rule. "Rounding down" can be confusing, since the digit doesn't change it only makes sense in a number-line context.
- Though it may not be expressed, a problem is sometimes that 47, say, seems much "closer" in form or name to 40 (it's got a 4 in the tens column) than it does to 50. This leads to confusion when asked "what's 47 closer to, 40 or 50?". The questioner means "closer on a number-line".

1.4.1	NEED stack of cards with different integer and decimal numbers (see sheets). Hold them up "What's this to the nearest integer?" etc. Or use acetate and point (see other sheet).	It doesn't matter if use the same cards again because you'll probably ask for a different degree of rounding. These cards (or the acetate) have multiple uses; e.g., "double these numbers", "find two numbers that add up to a certain amount", etc.
1.4.2	A less arbitrary/pointless task than rounding meaningless numbers is to use calculators to generate numbers to round. Interesting contexts include square roots and converting fractions to decimals (see sheets).	Answers are below.
1.4.3	If I've rounded a number to the nearest integer and I get 18, what's the smallest/biggest it could have been? The main difficulty is in seeing that 13.4999999999 is 13.5. Can use \geq and $<$ signs to clarify that the lower bound is the smallest number that will round up to the value whereas the upper bound is the smallest number that just won't round down to it.	Max and min possible values (upper and lower bounds). Pupils are often sceptical that 0.9999 = 1, but since $0.3333 = \frac{1}{3}$ (exactly, not approximately, so long as the 3's go on forever), you can multiply both sides by 3 and get 0.9999 = 1. (You can do a similar "trick" with $9 \times \frac{1}{9}$.)
1.4.4 1.4.5	NEED " $17 \div 5$ " sheet. Different answers to the same numerical question. It's useful to round the same number to different degrees of accuracy – hence a table is useful, say with ten numbers down the left side. This can be written on the board.	Sometimes it's best to be as accurate as possible, sometimes to round up, sometimes to round down. You can't guess from the size of a number how accurately it should be rounded; the context determines that.

number	to near- est int.	to near- est 10	to near- est 100	to near- est 1000
34624.51	34625	34620	34600	34000

Or you can have nearest integer, 1 dp, 2 dp, etc. or 1 sf, 2 sf, 3 sf, etc.

1.4.6 The trickiest cases are, e.g., 2499.7 rounded to the nearest integer. Can imagine rounding and carrying a 10 and then a hundred, but it may be easier to ask which two integers it's in between. Must be careful each time to round the original number and not the previous rounded answer (see 34620 in the table).

Answer: 2500 (it's in between 2499 and 2500 and nearer to 2500). Whenever rounding gets difficult, it's usually best to imagine or sketch a number-line.

ANSWERS

	1	2	3	4	5	6	7	8	9	10
1	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000
2	0.500	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000
3	0.333	0.667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	3.333
4	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000	2.250	2.500
5	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	2.000
6	0.167	0.333	0.500	0.667	0.833	1.000	1.167	1.333	1.500	1.667
7	0.143	0.286	0.429	0.571	0.714	0.857	1.000	1.143	1.286	1.429
8	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000	1.125	1.250
9	0.111	0.222	0.333	0.444	0.556	0.667	0.778	0.889	1.000	1.111
10	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000

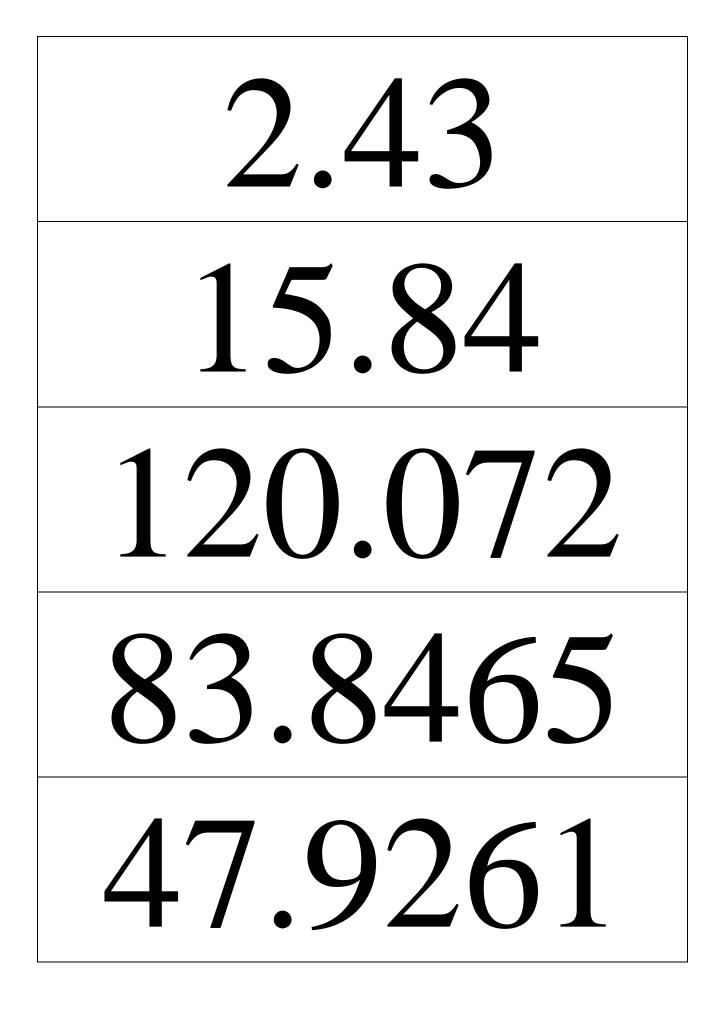
Fractions, Decimals and Rounding

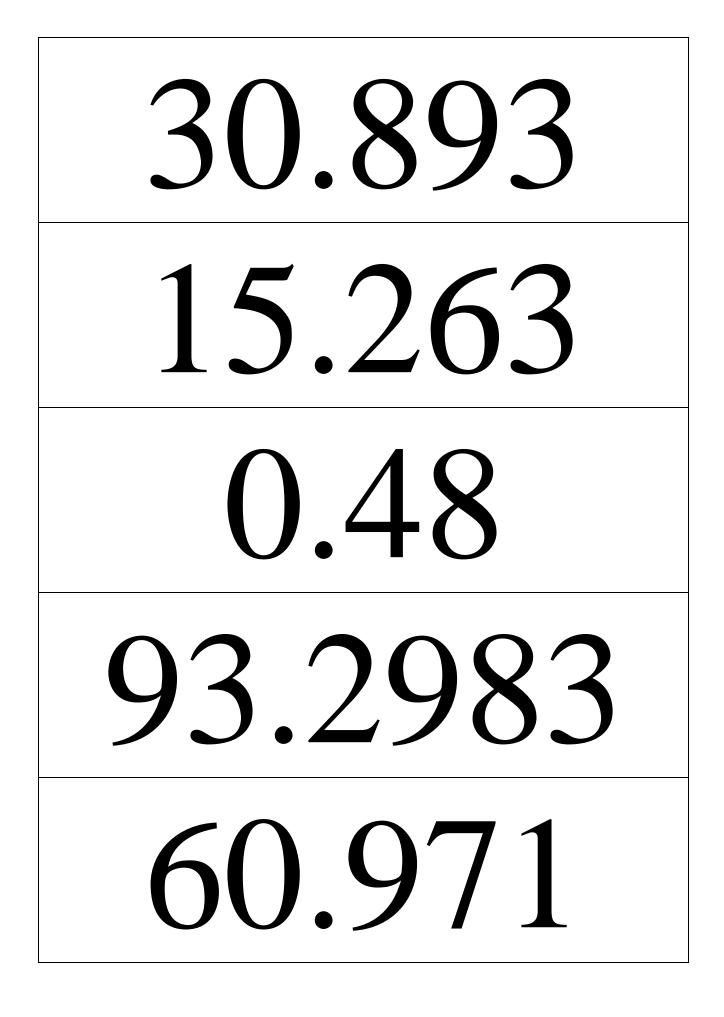
Rounding Square Roots

ANSWERS

x	\sqrt{x} (calculator display)	3 dp	2 dp	1 dp	nearest integer
1	1.00000000	1.000	1.00	1.0	1
2	1.414213562	1.414	1.41	1.4	1
3	1.732050808	1.732	1.73	1.7	2
4	2.00000000	2.000	2.00	2.0	2
5	2.236067977	2.236	2.24	2.2	2
6	2.449489743	2.449	2.45	2.4	2
7	2.645751311	2.646	2.65	2.6	3
8	2.828427125	2.828	2.83	2.8	3
9	3.000000000	3.000	3.00	3.0	3
10	3.162277660	3.162	3.16	3.2	3
11	3.316624790	3.317	3.32	3.3	3
12	3.464101615	3.464	3.46	3.5	3
13	3.605551275	3.606	3.61	3.6	4
14	3.741657387	3.742	3.74	3.7	4
15	3.872983346	3.873	3.87	3.9	4
16	4.000000000	4.000	4.00	4.0	4
17	4.123105626	4.123	4.12	4.1	4
18	4.242640687	4.243	4.24	4.2	4
19	4.358898944	4.359	4.36	4.4	4
20	4.472135955	4.472	4.47	4.5	4

The number of answers that round to n is simply 2n. So forty numbers round to 20.





47.6130.0270.6 243 l 313.50.182.5650 39.82 537.24291.30.027

Fractions, Decimals and Rounding

Look at the table below. We can use the numbers around the edge to make fractions.

Take the *horizontal* number as the **numerator** (top number of the fraction), and

the *vertical* number as the **denominator** (bottom number).

So the shaded box would be $\frac{4}{3}$ (or $1\frac{1}{3}$).

Use a calculator to convert this fraction to a decimal. Do this by working out the *numerator* divided by the *denominator*. So $4 \div 3 = 1.3333333...$ Round all your answers to *3 decimal places*. So we get 1.333

Complete the table.

Remember each time to do the *horizontal* number divided by the *vertical* number and to write all the answers to 3 decimal places.

	1	2	3	4	5	6	7	8	9	10
1										
2										
3				1.333						
4										
5										
6										
7										
8										
9										
10										

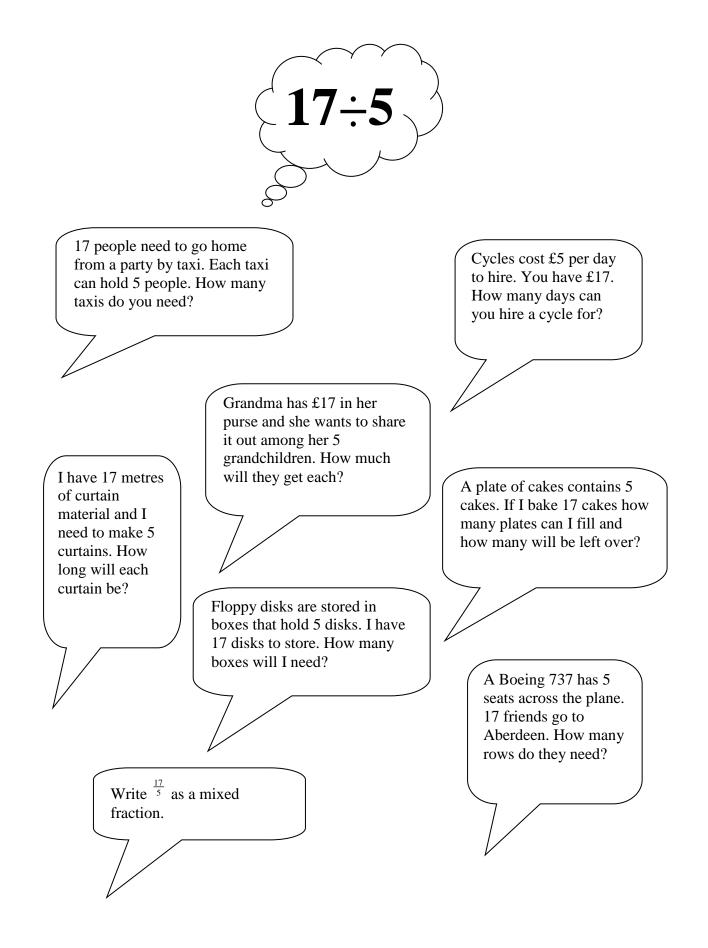
Rounding Square Roots

Use a calculator to find the square root of the number x each time. Round your answers to 3 dp, 2 dp, 1 dp and to the nearest integer. Round from the *original answer* each time and not from your previous rounding.

x	\sqrt{x} (as on calculator)	3 dp	2 dp	1 dp	nearest integer
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					

How many square roots are equal to 1 when rounded to the nearest integer? How many round to 2? How many round to 3?

Is there a pattern? How many do you think would round to 20?



Work out the answers to these problems. Are the problems really the same or different? Make up a different question. Which one is it most like?