1.6 Fractions

- Fractions are very unusual in having different equivalent forms. This concept takes some getting used to. Could go around pupil by pupil saying a fraction that's equivalent to a chosen starting fraction, and make a long line of = signs and equivalent fractions across the board. Those who find it easier can choose more complicated equivalent fractions. Once you get into a pattern (e.g., $\frac{30}{40}=\frac{300}{400}=\frac{30000}{40000}$), ask the next person to break the pattern and do something different.
- Rectangular "cakes" are generally easier to divide up than circular ones, and you can make use of squared paper. Also, the same cake can be cut in perpendicular directions to show a fraction of a fraction.
- Although we can *multiply* or *divide* the top and bottom of a fraction by the same number to get an equivalent fraction, it doesn't work if you *add* or *subtract* an amount; e.g., $\frac{2}{3}\neq\frac{3}{4}$ by adding 1 to top and bottom. This is a difficult concept to grasp – see section 1.10 for help with this.
- Common denominators make fractions easy to compare, but so do common *numerators*. For example, if asked which is bigger out of $\frac{2}{7}$ and $\frac{4}{15}$, working out a common denominator of 7×15 is unnecessary. Making the 1st fraction into $\frac{4}{14}$ makes it easy to see that this is bigger, since $\frac{1}{14}$ is more than $\frac{1}{15}$, so four of them must be bigger than four of the other. (Another way to compare the size of two fractions is to convert them both into decimals: you can think of this as effectively making a common denominator of 1.)

1.6.1 Number-lines are a good way to begin, as they make the point that fractions are just numbers on a number-line (like decimals).

Avoid allowing early work on fractions to be dominated by "fractions of*". They're not "operators", they're just numbers.*

1.6.2 Puzzle pictures (colour in the answers to produce a picture).

Often popular and available in books.

1.6.3 **NEED** Squared or isometric paper.
Draw shapes illustrating particular fractions; e.g., make a star shape and colour in $\frac{5}{8}$ of it.

Can make these into posters and use for display.

Pupils can make the shapes and the fractions as complicated as they like.

Differentiation by outcome.

1.6.4 Letters in words. What fraction of the word "letter" is "r", "t", "l", "e"? – total must be 1.
(Pupils can use their names for these.)

Answers: $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{6}$ and $\frac{1}{3}$. Easy to see that $\frac{2}{6}=\frac{1}{3}$ by grouping the letters in pairs. So it's useful to begin with words where repeated letters are adjacent "double-letters".

You can do a lot with this; e.g., "Find a word in which 'e' is $\frac{1}{2}$." Can make it easier by saying how many letters have to be in the word.

Answer: there are obviously many possible answers to these; e.g., "be", "been", etc.

This has a natural link to probability.

1.6.5 False Cancelling.
You can "cancel the 9's" in $\frac{19}{95}$ to get $\frac{1}{5}$, and these two fractions are the same size.
Find some others which work.

Answers:

$\frac{1\not{6}}{\not{6}4}=\frac{1}{4}$, $\frac{2\not{6}}{\not{6}5}=\frac{2}{5}$, $\frac{4\not{9}}{\not{9}8}=\frac{4}{8}=\frac{1}{2}$

1.6.6 **NEED** sets of cards containing different fractions (see pages – 1st page is easier; 2nd page is harder). In pairs, pupils shuffle the cards and place them face down on the table in a 4 × 4 grid.
Taking turns, each pupil turns over a pair of cards. If

If possible, photocopy each duplicate set onto a different coloured piece of card, so that different sets don't get muddled up, or write a different letter of the alphabet on the back of the cards in each different set, so pupils can check they have 16 "A" they are equivalent fractions he/she keeps them, otherwise they are turned back and left in the same positions. The winner is the one to collect the most pairs of cards.

cards (or whatever) before they start.

1.6.7 Find out how long a "number 3" haircut is.

Answer: each "number" corresponds to $\frac{1}{8}$ inch, so a "number 3" is supposed to be $\frac{3}{8}$ inch long.

1.6.8 There is a pile of sweets on the table. I come into the room and eat one sweet. I take a third of what's left and put them in my pocket for later.
The second person comes into the room and looks at the pile of sweets that's left. She eats 1, takes a third of the rest and puts them in her pocket and leaves the room. Finally, the third person comes into the room and does the same.
Afterwards, we divide the sweets that are left on the table equally among all three of us.
How many sweets were there to begin with?
How many do we each end up with?

(At every point we are always talking about a whole number of sweets.)

Answer: more than one possible answer; e.g., 25 sweets

25 → 24 → 16 → 15 → 10 → 9 → 6 → 2 each.

Each person has eaten 1. The 1st person has kept 8 + 2 = 10; the 2nd person has kept 5 + 2 = 7; the 3rd person has kept 3 + 2 = 5; and 3 + 10 + 7 + 5 = 25.

1.6.9 If I start with a fraction like $\frac{7}{38}$ and add 1 to the numerator and 1 to the denominator (so I end up with $\frac{8}{39}$), will the fraction get bigger or smaller?
Will this always happen?
Can you convince us why?

What if I add a number k to the top and bottom?
What if k is negative/decimal/fractional?

This can be useful when needing to compare, e.g., $\frac{13}{36}$ and $\frac{14}{37}$ to say which is bigger. The numerator has increased by a factor of $\frac{14}{13}$, whereas the denominator has increased by a factor of only $\frac{37}{36}$ (less), so $\frac{14}{37}$ must be bigger.

Answer: bigger
Many different ways of arguing this one. Could say that adding 1 to the 7 makes a proportionally bigger increase than does adding 1 to 38, so the numerator is increasing by a bigger proportion than the denominator. This will always happen with a "bottom-heavy" fraction like this.
With a "top-heavy" fraction, this process will make the fraction smaller for a similar reason.
The same argument will apply when any positive number is added.

Algebraically, a common denominator between $\frac{a}{b}$ and $\frac{a+k}{b+k}$ is $b(b+k)$, so $\frac{a}{b}=\frac{a(b+k)}{b(b+k)}$ and $\frac{a+k}{b+k}=\frac{b(a+k)}{b(b+k)}$ and $b(a+k)>a(b+k)$ because $b>a$ for a "bottom-heavy" fraction (assuming $k>0$).

For $a=b$, the two fractions are equivalent; e.g., $\frac{3}{3}=\frac{4}{4}=\frac{11}{11}$.

If $k<0$ it must not equal $-b$, otherwise the denominator will be zero.

Easier set – photocopy onto coloured card (could laminate it) and guillotine.

$\frac{1}{2}$	$\frac{5}{10}$	$\frac{1}{5}$	$\frac{2}{10}$
$\frac{1}{3}$	$\frac{2}{6}$	$\frac{7}{10}$	$\frac{14}{20}$
$\frac{1}{4}$	$\frac{5}{20}$	$\frac{1}{9}$	$\frac{3}{27}$
$\frac{3}{10}$	$\frac{30}{100}$	$\frac{2}{5}$	$\frac{6}{15}$

Harder set – photocopy onto coloured card (could laminate it) and guillotine.

$\frac{2}{3}$	$\frac{4}{6}$	$\frac{3}{4}$	$\frac{9}{12}$
$\frac{7}{11}$	$\frac{14}{22}$	$\frac{5}{9}$	$\frac{15}{27}$
$\frac{1}{3}$	$\frac{20}{60}$	$\frac{2}{19}$	$\frac{4}{38}$
$\frac{3}{22}$	$\frac{115}{110}$	$\frac{2}{7}$	$\frac{10}{35}$