

# 1.7 Fractions (Addition and Subtraction)

- Must be clear about the difference between the meaning of numerator numbers and denominator numbers. The numerator tells you how many there are. The denominator tells you what kind of thing you're talking about (like religious "denominations"). So in adding or subtracting fractions it doesn't make sense to add or subtract the denominators. (To do this is to treat fractions like column vectors, where both numbers in the vector represent

"similar" things and, e.g.,  $\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ .)

$\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$  is like  $\frac{2}{cars} + \frac{4}{cars} = \frac{6}{cars}$  or  $\frac{2}{sevenths} + \frac{4}{sevenths} = \frac{6}{sevenths}$  or  $\frac{2}{\pounds} + \frac{4}{\pounds} = \frac{6}{\pounds}$ , meaning £2 + £4 = £6.

It's worth spending time on this to make the point that the denominator is a kind of "label".

- Initially it's possible to allow *any* common denominator – having to find the LCM each time may over-complicate in the early stages.
- For addition, mixed numbers can be left as they are and the integers added separately. For subtraction, it's normally best to convert mixed numbers to "top-heavy" fractions, otherwise you have to deal in negative fractions in cases like  $2\frac{1}{5} - 1\frac{3}{4}$ . To convert a mixed number, say  $1\frac{2}{7}$ , to a "top-heavy" fraction, think of it as  $1 + \frac{2}{7} = \frac{7}{7} + \frac{2}{7} = \frac{9}{7}$  and similarly (going the other way) to convert a "top-heavy" answer into a mixed number. Digits next to each other always mean addition (although adjacent *letters*, or number and letters, in algebra always mean multiplication); e.g., 34 means 30+4 and 4.8 means 4+0.8, so  $1\frac{2}{7}$  means  $1 + \frac{2}{7}$ .
- It's still worth using number-lines to reinforce that addition and subtraction are still just moving forwards and backwards along a normal number-line.

**1.7.1** One way to begin by building on an awareness of equivalent fractions is to scatter some "random" fractions on the board and ask pupils in groups to pick pairs of fractions that they think will add up to *less than 1*. They should try to justify their choices to one another.

*For example, pupils may say that  $\frac{1}{2} + \frac{1}{5} < 1$ .*

*because  $\frac{1}{2} + \frac{1}{2} = 1$  and  $\frac{1}{5} < \frac{1}{2}$ .*

*Another might be that  $\frac{3}{4} + \frac{1}{5} < 1$  because*

*$\frac{3}{4} + \frac{1}{4} = 1$  and  $\frac{1}{5} < \frac{1}{4}$ .*

*More sophisticated arguments are also possible.*

*There may be certain pairs left over that it isn't possible to decide about – so we need a way of working out the total of two fractions.*

Can you find more than two fractions whose total is less than 1?

**1.7.2** Puzzle pictures (colour in the answers to produce a picture).

**1.7.3** It is possible to use the digits from 1 to 9 once each to make fractions that add up to 1. How do you do it?

$$\frac{2}{12} + \frac{7}{34} + \frac{2}{68} = 1$$

*A collection of fractions might be the following (with their sums given in the grid). But write them onto the board in no particular order.*

	$\frac{1}{5}$	$\frac{2}{9}$	$\frac{4}{11}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
$\frac{1}{5}$	$\frac{2}{5}$							
$\frac{2}{9}$	$\frac{19}{45}$	$\frac{4}{9}$						
$\frac{4}{11}$	$\frac{31}{55}$	$\frac{58}{99}$	$\frac{8}{11}$					
$\frac{1}{2}$	$\frac{7}{10}$	$\frac{13}{18}$	$\frac{19}{22}$	1				
$\frac{3}{5}$	$\frac{4}{5}$	$\frac{37}{45}$	$\frac{53}{55}$	$1\frac{1}{10}$	$1\frac{1}{5}$			
$\frac{2}{3}$	$\frac{13}{15}$	$\frac{8}{9}$	$1\frac{1}{33}$	$1\frac{1}{6}$	$1\frac{4}{15}$	$1\frac{1}{3}$		
$\frac{3}{4}$	$\frac{19}{20}$	$\frac{35}{36}$	$1\frac{5}{44}$	$1\frac{1}{4}$	$1\frac{7}{20}$	$1\frac{5}{12}$	$1\frac{1}{2}$	
$\frac{4}{5}$	1	$1\frac{1}{45}$	$1\frac{9}{55}$	$1\frac{3}{10}$	$1\frac{2}{5}$	$1\frac{7}{15}$	$1\frac{11}{20}$	$1\frac{3}{5}$

*Here there are sixteen pairs (including two the same) which add up to less than 1 (shaded above).*

*Many possibilities.*

*Often popular and available in books.*

*Answer:*  $\frac{9}{12} + \frac{5}{34} + \frac{7}{68} = 1$

Also, using zero as well,  $\frac{35}{70} + \frac{148}{296} = 1$  and  $96 + \frac{2148}{537} = 100$  and others.

1.7.4 Magic squares with fractions (see sheet).

1.7.5 Patterns in adding fractions.

Work out  $\frac{1}{2} + \frac{1}{4}$ , then  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ , then

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ , and so on.

What is happening?

Try a different pattern, still dividing by a constant amount each time;

e.g.,  $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

What does the total get closer and closer to?  
Can you summarise what will happen?

What if you started with  $\frac{1}{10}$ ?

1.7.6 Work these out.

What do you notice?

$$\frac{1+3}{5+7}, \frac{1+3+5}{7+9+11}, \frac{1+3+5+7}{9+11+13+15}$$

1.7.7 Find the sums of interesting-looking sequences;

e.g.,  $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$  or even

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10}.$$

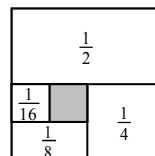
(The second one is too big for most calculators to handle!)

Photocopy the sheet or copy some onto the board.

Answer:

The total  $(\frac{3}{4}, \frac{7}{8}, \frac{15}{16})$  gets closer and closer to 1.

It's easy to see from the drawing below that the total area is approaching the  $1 \times 1$  square.



In general for a geometric series  $\sum_{n=1}^{\infty} \left(\frac{1}{x}\right)^n = \frac{1}{x-1}$  for

$|x| > 1$ , so if  $x = 3$ , then the sum approaches  $\frac{1}{2}$  as you add on more and more terms.

And with  $\frac{1}{10}$  the infinite sum is  $\frac{1}{9}$ .

Answer: all equal  $\frac{1}{3}$  (the fraction that would fit at the beginning of the pattern).

Galileo (1564-1642) noticed this.

Answers:  $1 \frac{83}{140}$  and  $2 \frac{2341}{2520}$ .

(There's no elegant short-cut like there is with adding the integers from 1 to 10 to get 55.)

After doing all the work involved in these, you're unlikely ever to think again that adding fractions is a simple matter of adding the numerators and adding the denominators!

## Fractions Calculations

1. When *adding or subtracting* fractions, you always need to find a *common denominator*.
2. When *multiplying* fractions, you *don't* need to find a common denominator, but you should always *cancel* as much as possible before *multiplying the numerators (tops)* and *multiplying the denominators (bottoms)*.
3. When *dividing* fractions, leave the *first* fraction alone, turn the *second* fraction *upside down* and change the sign to *multiply*.
4. Change *mixed numbers* into "top-heavy" fractions for subtracting, multiplying and dividing.
5. Always *simplify* your final answer and turn "top-heavy" fractions into *mixed numbers*.

## *Magic Squares (Fractions)*

Fill in the missing fractions in these *magic squares*.

In each square the total along every column, every row and both diagonals is the same. Different squares have different totals. Write all the fractions in their simplest forms.

1

$\frac{4}{15}$		
$\frac{1}{5}$	$\frac{1}{3}$	
$\frac{8}{15}$		

2

$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{25}$
	$\frac{13}{100}$	

3

$1\frac{2}{9}$		
	1	$\frac{5}{9}$
		$\frac{7}{9}$

4

5		
$6\frac{2}{3}$	$4\frac{1}{3}$	1

5

	$1\frac{4}{7}$	
	1	
	$\frac{3}{7}$	$1\frac{1}{7}$

6

		5
	$6\frac{1}{2}$	
8		11

## Magic Squares (Fractions)

## ANSWERS

Fill in the missing fractions in these *magic squares*.

In each square the total along every column, every row and both diagonals is the same.

Different squares have different totals. Write all the fractions in their simplest forms.

### 1 magic total 1

$\frac{4}{15}$	$\frac{9}{15}$	$\frac{2}{15}$
$\frac{1}{5}$	$\frac{1}{3}$	$\frac{7}{15}$
$\frac{8}{15}$	$\frac{1}{15}$	$\frac{6}{15}$

### 2 magic total $\frac{39}{100}$

$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{25}$
$\frac{7}{100}$	$\frac{13}{100}$	$\frac{19}{100}$
$\frac{11}{50}$	$\frac{1}{100}$	$\frac{4}{25}$

### 3 magic total 3

$1\frac{2}{9}$	$\frac{1}{9}$	$1\frac{6}{15}$
$1\frac{4}{15}$	1	$\frac{5}{9}$
$\frac{1}{3}$	$1\frac{8}{15}$	$\frac{7}{9}$

### 4 magic total 9

5	$1\frac{2}{3}$	$2\frac{1}{3}$
$\frac{1}{3}$	3	$5\frac{2}{3}$
$6\frac{2}{3}$	$4\frac{1}{3}$	1

### 5 magic total 3

$\frac{6}{7}$	$1\frac{4}{7}$	$\frac{4}{7}$
$\frac{5}{7}$	1	$1\frac{2}{7}$
$1\frac{3}{7}$	$\frac{3}{7}$	$1\frac{1}{7}$

### 6 magic total $19\frac{1}{2}$

2	$12\frac{1}{2}$	5
$9\frac{1}{2}$	$6\frac{1}{2}$	$3\frac{1}{2}$
8	$\frac{1}{2}$	11