1.10 Ratio

- Ratio can be a comparison between 2 quantities or more than 2 quantities. Recipes are a possible starting point; e.g., pancakes (4 tbsp plain flour, pinch of salt, 1 egg, 300 ml milk). How much mixture could you make with 1 pint (about 600 ml) milk? How many eggs would you need, etc.? An example like this can destroy "linear thinking": "500g flour and 1 egg" – if I wanted to make a larger amount would I really use "501g flour and 2 eggs"? or "600g flour and 101 eggs"?! Why doesn't this work?
- Fizzy orange: You can buy orangeade in the shops but it tastes much better if you make it yourself out of real orange juice from a carton and lemonade. Imagine we're making some for this class. How much do you think we would need? 6 litres? Suppose I use 2 litres of orange with 4 litres of lemonade. Imagine that isn't going to be enough, and I want 8 litres, so instead I use 3 litres of orange with 5 litres of lemonade? (Write up these possibilities on the board – see table below.) Will it taste the same?
  (Some pupils may argue that it will because you've added the same extra amount of orange as lemonade.) If not, will it taste more 'orangey' or more fizzy? Be awkward – "Are you saying it will taste more orangey because it's got more orange in it? But it's got more lemonade in it as well."
  Answer: 2nd one more orangey because $\frac{3}{8} > \frac{1}{3}$. *Expect arguments like this: "You've added the same amount of lemonade as orange but it has less effect because there was more lemonade to start with. Imagine 1 litre of orange with 99 litres of lemonade. 1 more litre of orange will make it nearly twice as orangey but one more litre of lemonade won't make any noticeable difference."*
  If pupils start talking about 3 litres of orange and 6 litres of lemonade, or other possibilities, write them up in a table like this:

	A	***B***	***C***
orange	*2*	*3*	*3*
lemonade	*4*	*5*	*6*
total	*6*	*8*	*9*

  When you've got A, B, C, D, E, etc., decide which will taste the same. Could make a scale of "orangeyness" going one way and fizziness going the other way and decide where to put A, B, C, …
  This can be a very useful context because it's clear what is the same (the taste) when the "ratio" is the same. This is also true with the next example (colour).
- Paint: I've made light blue paint by mixing 1 tin of white with 2 tins of blue. I need 1 more tin of light blue. How much blue and white paint should I use to make it exactly the same shade of blue?
  Answer: $\frac{1}{3}$ *tin of white with* $\frac{2}{3}$ *tin of blue.*
- Beads on a necklace – red and green: RRGGGRRGGGRRGGG. Ratio can involve discrete items as well as continuous amounts.

1.10.1 Scaling up recipes.
Cake: 4 oz (100 g) flour; 4 oz (100 g) sugar; 4 oz (100 g) margarine; 2 eggs.
Pastry: 8 oz (225 g) flour; 2 oz (50 g) lard; 2 oz (50 g) margarine; 2 tbsp water.
Biscuits: 8 oz (225 g) flour; 4 oz (100 g) sugar; 4 oz (100 g) margarine; 1 egg.
Shortbread: 9 oz (250 g) flour; 3 oz (75 g) sugar; 6 oz (175 g) butter.

What about the oven temperature and the cooking time?

In fact for a larger quantity you might go for a lower temperature and longer cooking time.

Imagine making enough for the whole class. Generally that will be an awkward number, which raises the issue of how accurate we need to be ("5.6667 pinches of salt", etc.!).

They may need to be higher/longer for a larger recipe, but you certainly don't want to scale them up in the same proportion as the ingredients! (Scaling up temperature is a nonsense anyway, as it makes a difference whether you use °C or °F!) See if pupils can explain why it's silly. Could imagine two identical ovens next to each other in the kitchen cooking identical cakes. Then combine into one big cake in one big oven.

1.10.2 Puzzle pictures (colour in the answers to produce a picture).

Often popular and available in books.

1.10.3 It takes $1\frac{1}{2}$ hens $1\frac{1}{2}$ days to lay $1\frac{1}{2}$ eggs. How long

Answer: 3 days (since 2 hens between them lay 2

does it take 2 hens to lay 4 eggs?

Many problems can be solved in a similar way, using a table like this.

eggs every $1\frac{1}{2}$ days).

Can use a table to make it clear, always keeping 1 thing the same as you go to the next line.

hens	***days***	***eggs***
$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$
1	$1\frac{1}{2}$	1
2	$1\frac{1}{2}$	2
2	**3**	4

1.10.4 To eat a bowl of porridge it takes
a Scotsman 2 minutes;
an Englishman 4 minutes (less expert!);
a Welshman 8 minutes; and
an Irishman also 8 minutes.
If they all share a bowl of porridge (but have a spoon each) how long will it take all of them together? (We have to assume they don't slow each other down!)

Answer: 1 minute
Hint questions: "Will it take more or less than 8 minutes?" "More or less than 2 minutes?"
One way is to think about porridge-eating-rates.
S = $\frac{1}{2}$ a bowl per minute
E = $\frac{1}{4}$ of a bowl per minute
W = I = $\frac{1}{8}$ of a bowl per minute
So in 1 minute they finish off the whole bowl because $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=1$.

1.10.5 A goat takes 3 minutes to eat a cabbage.
A rabbit takes 4 minutes and a mouse takes 5. If a single cabbage were thrown to all three animals and they ate together, how long would they take between them to eat the whole cabbage?
What do you have to assume?

Answer: In 1 minute, they'd eat $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}=\frac{47}{60}$ of the cabbage, so in $\frac{60}{47}=1\frac{13}{47}$ minutes, or 1 minute 17 seconds, they'd manage the whole thing.

That they don't fight or get in each other's way or try to eat each other!

1.10.6 Investigate A2, A3, A4, A5, A6 paper.
What is special about "A-size" paper?
Why does this happen?

Answer: Long side to short side is $\sqrt{2}:1$, so that cutting in half (halving the area) gives a piece of paper the same shape (sides in same ratio).
This follows from saying that $x:1=1:\frac{1}{2}x$ so that $\frac{x}{1}=\frac{1}{\frac{1}{2}x}$ or $\frac{1}{2}x^2=1$, so that $x=\pm\sqrt{2}$, but only the positive solution makes sense here.

1.10.7 A script for a TV sit-com has about 6000 words per $\frac{1}{2}$ hour of running time. What is this in words per second?

Answer: about 3 (surprisingly, perhaps!) – not many pauses in dialogue during a TV sit-com!

1.10.8 Old-fashioned school maths questions; e.g., "If 200 men working 8 hours a day take 12 days to dig a trench 160 yards long, 6 yards wide and 4 yards deep, how many days will it take 90 men, working 10 hours a day to dig a trench 450 yards long, 4 yards wide and 3 yards deep?"

Answer: $12\times\frac{8}{10}\times\frac{200}{90}\times\frac{450}{160}\times\frac{4}{6}\times\frac{3}{4}=30$ *days.*

These are not too hard so long as you don't panic and just think about whether each factor will make the job take more time or less time.

1.10.9 Value for money for products from the supermarket (either £ per g or g per £).

Could visit a supermarket. (Or bring in empty packets or special offer adverts.)

1.10.10 If x people can pack 20 bags in x minutes, how long will it take $x+2$ people to pack 20 identical bags?

What assumption do you make?

Answer is not $x+2$ minutes because it will take less *time with more people.*
Rate of working is $\frac{20}{x^2}$ bags per minute per person, so with $x+2$ people it will be, $\frac{20(x+2)}{x^2}$ bags per minute, so the time for 20 bags will be $20\div\frac{20(x+2)}{x^2}=\frac{x^2}{x+2}$ *minutes.*

We assume that with more people you get proportionately more work, and not just more chat!

(Alternatively, it will take $\frac{x}{x+2}$ of the time it took x people, so that is $\frac{x}{x+2} \times x$ which is $\frac{x^2}{x+2}$ minutes.)

1.10.11 If you have a bottle of drink worth £10 and another worth only £5, how much of each would you need to mix to make 1 bottle worth £8?

Answer: $\frac{3}{5}$ of the £10 bottle and $\frac{2}{5}$ of the £5 bottle, because $\frac{3}{5} \times 10 + \frac{2}{5} \times 5 = 8$.
(Trial and improvement or solve simultaneously $10a + 5b = 8$ and $a + b = 1$.)

1.10.12 A painter mistakenly mixes 5 litres of white paint with 3 litres of blue paint, when he meant to do it the other way round. How can he get the amount and colour that he wanted with the minimum wastage of paint?
(This would be more complicated if the different colours cost different amounts.)

Answer: He has used too much white paint, so he won't need to use any more of that. He needs to pour off $\frac{2}{5}$ of the mixture (= $\frac{2}{5} \times 8$ = 3.2 litres) leaving 3 litres of white and 1.8 litres of blue. Then add 3.2 litres of blue. This wastes an additional 3.2 litres only.

1.10.13 I have 50 cm^3 orange juice in one glass and 50 cm^3 water in another glass. I take 1 cm^3 of orange juice from the first glass and add it to the second. After stirring the second glass thoroughly I take 1 cm^3 of it and add it to the first glass (so both glasses again contain 50 cm^3). Is there more orange juice in the second glass or more water in the first?

Answer: The same, since the total volume in each glass is 50 cm^3, and it's either in one glass or the other: stirring thoroughly doesn't affect that.
In more detail, going stage by stage,

first glass		***second glass***	
orange	***water***	***orange***	***water***
50	0	0	50
49	0	1	50
$49\frac{1}{51}$	$\frac{50}{51}$	$\frac{50}{51}$	$49\frac{1}{51}$

So there is $\frac{50}{51}$ cm^3 of each in the other if thoroughly mixed.

1.10.14 A traffic warden takes 3 minutes 10 seconds to write out a parking ticket for a car. If he works at the same rate all afternoon, how long would it take him to make 60 similar bookings?

Answer: 3 hours, 10 minutes.
(Multiplying minutes and seconds by 60 gives hours and minutes.)

1.10.15 Gold. What does "24 carat" gold mean?
(Pupils could find out for homework, perhaps.)

1 carat = $\frac{1}{24}$ of the total mass, so 24 carat gold is 100% pure. For most purposes this is too soft, so copper or silver are commonly added.
18 carat gold is $\frac{18}{24} = \frac{3}{4}$ gold.

What about diamonds?
What's the maximum carat?

With diamonds, carats measure the mass of the diamond. 1 carat = 200 mg, so there's theoretically no limit to the number of carats.

1.10.16 Petrol.
A petrol pump delivers fuel at a rate of 25 litres per minute. How much money is it costing per second? (Use current petrol price; e.g., 75 p per litre of unleaded fuel.)

Answer: With this figure, it comes to about 30 pence per second.