1.11 Fractions, Decimals and Percentages

- Fractions-decimals-percentages triangle. Can go clockwise or anticlockwise (see below).
  When going from fraction to %, if the denominator is a factor of 100, it's easiest simply to convert the fraction to something out of 100. But if the denominator isn't a factor of 100, then go round the triangle the other way *via* the decimal equivalent.
- The trickiest conversion is from a decimal into a fraction. The easiest way may be to write the decimal as a fraction "over 1" and then multiply numerator and denominator by 10 (or powers of 10) until both numbers are integers; then simplify as normal; e.g., 0.38 $= \frac{0.38}{1} = \frac{38}{100} = \frac{19}{50}$.

1.11.1 Three parallel number-lines can show the inter-conversion between fractions, decimals and % (go from 0 but beyond 1 to show that % can be greater than 100).

This reinforces that these are just three different ways of writing numbers.

1.11.2 **NEED** cards for "chain game" (see sheet of cards to use). Hand out all the cards (1 to each pupil, 2 or even 3 to some or do 1 or more yourself – you must use all the cards).
Locate the person who has the "start" card. He/she begins and reads out their card, and whoever has the answer reads theirs and so on.

Could time it – "Can we do it in less than $1\frac{1}{2}$ minutes?" – although beware of this putting too much pressure on one or two individuals.

(With a small class you may need to cover several cards yourself or make use of a teaching assistant to do that.)

1.11.3 Puzzle pictures (colour in the answers to produce a picture).

Often popular and available in books.

1.11.4 It's easy to make a table on the board with gaps for pupils to fill in (either on the board or on paper).

fraction	decimal	percentage
	0.7	
$\frac{4}{5}$		

Can throw in a recurring decimal as a challenge (see section 1.11.5 below).

The hardest ones are usually, e.g., converting 0.1% to 0.001 or 2 to 200%.

1.11.5 Converting recurring decimals to fractions; e.g., 0.4444…; 0.424242…; 0.123451234512345…; 0.007007007007…

Answers: $\frac{4}{9}$, $\frac{42}{99}$, $\frac{12345}{99999}$ *and* $\frac{7}{999}$.
You put the recurring part in the numerator and always as many 9's in the denominator as digits in the recurring section.
Of course, $\frac{3}{9} = \frac{1}{3}$ *(0.3333…), so that fits the pattern.*

Numbers greater than 1 are no more difficult; e.g., 1.1111111… or 1.23123123123123… or 1.123123123123123… or 3.555555555…

$1\frac{1}{9} = \frac{10}{9}$, $1\frac{231}{999} = \frac{1230}{999}$, $1\frac{123}{999} = \frac{1122}{999}$, $3\frac{5}{9} = \frac{32}{9}$.

Another method is to write, say, $0.2222... = \frac{2}{10} + \frac{2}{100} + \frac{2}{1000} + ...$ *and to sum this as an infinite geometric series using* $\sum_{n=1}^{\infty} \frac{a}{x^n} = \frac{a}{x-1}$ *to give* $\frac{2}{10-1} = \frac{2}{9}$.

These can also be solved by letting x *be the fraction you're trying to find and writing, say,*
$x = 0.7373737373...$
$100x = 73.7373737373...$ *and subtracting gives*
$99x = 73$ *so* $x = \frac{73}{99}$.

1.11.6 Investigate recurring digits in decimal expansions of fractions; e.g., which fractions have recurring decimals and which don't, how long the recurring

Answers: denominators which take the form $2^x 5^y$ *where* x *and* y *are integers* ≥ 0*; e.g., halves, quarters, tenths, twentieths, etc. give terminating* unit is, etc.

decimals.
If the denominator is n then the length of the recurring part of the decimal must be $\leq (n-1)$.
Beyond that it isn't easy to generalise, except that once the fraction is in its simplest form the length of the recurring unit depends only on the denominator and not on the numerator.

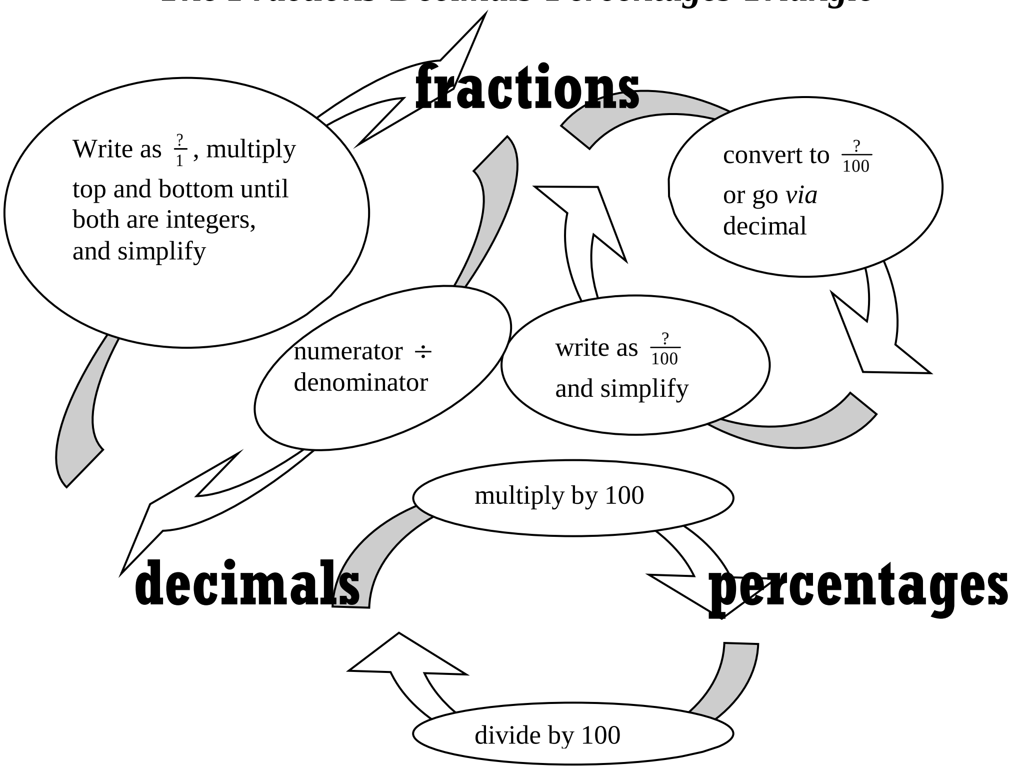
1.11.7 Comparing the size of similar fractions.
Using consecutive pairs of terms from the Fibonacci series (especially high terms) as the numerator and denominator makes fractions that are similar in size (approaching the golden ratio 1.618… as you take higher and higher terms).

e.g., the Fibonacci sequence begins 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …
So $\frac{2}{3} > \frac{5}{8}$ and $\frac{8}{13} < \frac{13}{21}$, etc.

"Making the denominators the same" is a good method; in fact, although we may not think of it that way, that's what we're doing when we convert both of the fractions into decimals.
e.g., $\frac{8}{13} = \frac{8 \div 13}{13 \div 13} = \frac{0.6154}{1}$ *(4 dp),*
and $\frac{13}{21} = \frac{13 \div 21}{21 \div 21} = \frac{0.6190}{1}$ *(4 dp).*
Denominators are both 1 now.
(Converting to both to percentages is making the denominators both 100.)

But making numerators the same is just as good. For example, to compare $\frac{2}{7}$ *and* $\frac{4}{15}$*, working out a common denominator of* 7×15 *is unnecessary. Instead we can make the first fraction into* $\frac{4}{14}$ *and then we can see that this is bigger than* $\frac{4}{15}$*, since* $\frac{1}{14}$ *is more than* $\frac{1}{15}$*, so four of them must be bigger than four of the other.*

The Fractions-Decimals-Percentages Triangle



I am … start	I am … $\frac{7}{10}$	I am … 0.25	I am … 20%
You are … 0.7	You are … 25%	You are … $\frac{1}{5}$	You are … 90%
I am … $\frac{9}{10}$	I am … $\frac{3}{5}$	I am … 28%	I am … 0.75
You are … 0.6	You are … 0.28	You are … $\frac{3}{4}$	You are … 1%
I am … $\frac{1}{100}$	I am … 73%	I am … 0.1	I am … $33\frac{1}{3}\%$
You are … 0.73	You are … $\frac{1}{10}$	You are … $\frac{1}{3}$	You are … 0.41
I am … $\frac{41}{100}$	I am … 88%	I am … 0.81	I am … $\frac{2}{5}$
You are … 0.88	You are … $\frac{81}{100}$	You are … 40%	You are … 0.99

Photocopy onto coloured card (could laminate it) and guillotine along the **bold** lines only.
Both sheets together make a whole set – 32 cards should be enough for most classes.

I am … $\frac{99}{100}$	I am … 0.17	I am … 62%	I am … 0.04
You are … 17%	You are … $\frac{31}{50}$	You are … $\frac{1}{25}$	You are … 6%
I am … $\frac{3}{50}$	I am … 11%	I am … 0.35	I am … 0.05
You are … 0.11	You are … $\frac{7}{20}$	You are … 5%	You are … $\frac{3}{10}$
I am … 30%	I am … $\frac{37}{50}$	I am … $\frac{3}{25}$	I am … 9%
You are … 0.74	You are … 12%	You are … 0.09	You are … $\frac{13}{50}$
I am … 0.26	I am … 0.38	I am … 45%	I am … $\frac{1}{2}$
You are … 38%	You are … $\frac{9}{20}$	You are … 50%	You are … stop

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