

1.15 Standard Form

- Confident calculator use is very important, using the x^y or y^x or $^$ buttons for powers, but the EXP button for standard form (and *not* typing in the 10 or the \times); i.e., for 6×10^4 typing “6 EXP 4” only.
- One approach to positive and negative indices is to say that $\times 10^5$ means multiply by 10 “five times” ($\times 10 \times 10 \times 10 \times 10 \times 10$), whereas $\times 10^{-5}$ means divide by 10 “five times” ($\div 10 \div 10 \div 10 \div 10 \div 10$).

1.15.1 Science provides nice contexts for very big and very small numbers; could look at the prefixes used (see sheet).

1.15.2 Standard Form. You can introduce this by writing the mass of the earth (6×10^{24} kg) and the mass of Jupiter (2×10^{27} kg) with all their zeroes on the board.

Deliberately squash up the zeroes in Jupiter to make the number look shorter.

Which planet do you think is “heavier”?

We need a better way of writing very big (and very small) numbers. You could write up the mass of an electron (9.1×10^{-31} kg) in decimal form – not easy to read or write like that.

1.15.3 People are used to decimal notation for large numbers; e.g., £2.3m for 2.3 million pounds. You can think of the 10^3 in 4.2×10^3 , say, as either a multiplier (start with 4.2 and move all the digits 3 places to the left) or as a place value column heading (the 4 goes into the 10^3 column; i.e., the thousands column).

1.15.4 **NEED** *Guinness Book of Records* (or similar). Find numbers that you could write in standard form.

1.15.5 Work out how many seconds you’ve been alive, how many seconds till we go home at the end of term (do we finish early on the last day?), how many seconds of holiday till we come back, how many seconds till 12 noon on Christmas day or the new year, how many seconds till your birthday, how many seconds of maths lessons we have each week/fortnight, etc.

1.15.6 Estimate how many water molecules you think there are in the swimming pool.

What do you need to know to work it out?

18 g of water (1 mole) contains Avogadro’s number (6×10^{23}) molecules.

The density of water is 1 g/cm³.

Large: space, galaxies, etc.

Small: atoms, molecules, etc.

(see sheet)

Answer: Pupils will probably still say Jupiter because they’ve seen pictures/diagrams that show it bigger – but bigger size doesn’t necessarily mean greater mass.

Pupils will probably realise the attempted deception!

The mass of the sun is 2×10^{30} kg.

Could use this for a homework – pupils could use a newspaper or magazine or the internet if they don’t have the book.

Answers will obviously vary:

For a 13-year-old, seconds alive is about $13 \times 365 \times 24 \times 60 \times 60 = 4.1 \times 10^8$.

It can be interesting to try to estimate the answers before working them out. We don’t have a “feel” for size when we’re in units that seem too small or too big.

Answer:

Could consider the sloping bottom of the pool and treat the water as a trapezoidal prism, or just use average depth (e.g., 1.5 m);

e.g., take the pool as 50 m by 25 m by 1.5 m deep, so volume = $50 \times 25 \times 1.5 = 1875 \text{ m}^3$

= $1.9 \times 10^9 \text{ cm}^3 = 1.9 \times 10^9 \text{ g}$, so it contains

$1.9 \times 10^9 \times 6 \times 10^{23} \div 18$ molecules

= 6×10^{31} molecules (approx).

Large and Small Numbers

prefix	symbol	size
tera-	T	10^{12}
giga-	G	10^9
mega-	M	10^6
kilo-	k	10^3
hecto-	h	10^2
deka-	D	10^1
<i>basic unit</i>		10^0
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}
atto-	a	10^{-18}

1 kilobyte is more than 1000 bytes – actually 1024 bytes because $2^{10} = 1024$

Distances – large and small	
10^{26} m	most distant galaxies
10^{21} m	size of our galaxy
10^{11} m	earth-sun distance
10^9 m	sun (diameter)
10^8 m	Jupiter (diameter)
10^7 m	earth (diameter)
10^4 m	black hole
10^{-4} m	smallest object visible to naked eye
10^{-6} m	smallest object visible under light microscope
10^{-8} m	large molecules
10^{-10} m	atoms
10^{-14} m	nucleus of atom
10^{-15} m	proton or neutron

10^{16} m = 1 light year (approx)
 (speed of light = 3×10^8 m/s, and a light year is the distance light travels in a vacuum in a year, so 1 light year
 = $3 \times 10^8 \times 60 \times 60 \times 24 \times 365$ metres)

US billion (10^9) used even by Bank of England nowadays; old UK billion (10^{12}) obsolete.

Names (and number of zeroes in brackets)

billion (9), trillion (12), quadrillion (15), quintillion (18), sextillion (21), septillion (24), octillion (27), nonillion (30), decillion (33), undecillion (36), etc.

googol (100); and a googolplex has a googol of zeroes (so it's $10^{(10^{100})}$).

1 googol is more than the total number of protons, neutrons and electrons in the known universe!

milliard = billion (10^9) – useful when billion could mean 10^9 or 10^{12}

lakh (10^5) and crore (10^7) are used in South Asia

1 million microphones = 1 phone
 1 million phones = 1 megaphone
 10 cards = 1 decacards
 1 millionth of a fish = 1 microfish
 10 millipedes = 1 centipede
 10^{12} pins = 1 terrapin