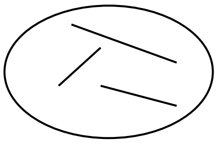
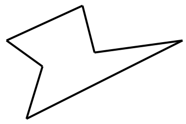
2.1 Polygons

- A topic containing lots of definitions. One way to make this interesting is for pupils to look for "hard cases" that get around other people's definitions. Or the teacher can do that at the board as the pupils attempt to define key concepts;

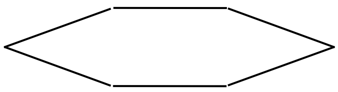


e.g., "A polygon is a shape containing straight lines", so the teacher draws something like this (left), and the pupils have to think of a better definition. "I'm going to be awkward – try and come up with a definition I won't be able to get around." You might eventually end up with something like "a flat closed shape made up entirely of straight sides", or better.

- It's worth emphasising that odd-looking, non-standard polygons (e.g., see right), are still polygons (it's even a hexagon), and that there's nothing "wrong" with them.



- "Regular" means that all the sides have the same length *and* all the angles are equal (or "all the vertices look the same" if "angle" is not yet a clear concept). It's helpful to see that both of these conditions must hold by imagining irregular hexagons like those below.

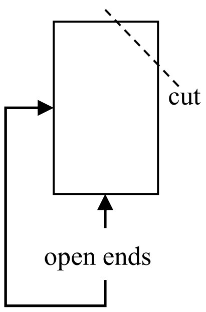


all sides the same length, but angles different sizes

all angles the same size, but sides different lengths

So the only *regular* quadrilateral, for instance, is the square (e.g., you can't have a "regular trapezium", etc.).
- Material involving angles in polygons is in section 2.4.

2.1.1 **NEED** newspaper, scissors and practice!
Fold a whole sheet of newspaper in half and then in half again. The teacher then cuts a shape out of the corner which corresponds to the centre of the original sheet.

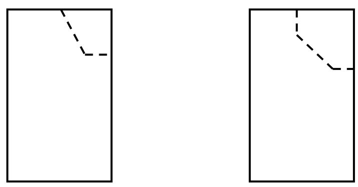


"When I open it out, what will we see?"
A hole in the middle.
"What shape will the hole be?"

Then open and see.
"What shape is it?"

Pupils can use scrap paper or newspaper to try to make particular shapes.
You could discuss reflection symmetry.

Answers:
Ask pupils to predict each time before you open out the newspaper.
1. *Cut a straight line at 45° to both edges to get a* ***square****. (Expect pupils to say "diamond" because of the orientation.)*
2. *A similar cut not at 45° makes a* ***rhombus****.*
3. *Cut lines at 120˚ in a 2:1 ratio as below (left) to make a* ***regular hexagon****.*
4. *Cut lines at 135˚ in a 1:2:1 ratio as below (right) to make a* ***regular octagon****.*



It's possible to make many other shapes by experimenting.

2.1.2 "I spy a polygon!". Looking for polygons in the classroom or around the school.
So long as they don't have to be regular, there should be lots. You can declare that "squares and rectangles are boring".

This can be a brief task at the beginning or end of a lesson.

You can offer challenges such as "Can anyone see a heptagon?"

2.1.3 Names of polygons. Make a table (see sheet).
Discuss how pupils are going to remember the

1- and 2-sided polygons don't exist; the only special names for regular *polygons are "equilateral*

names.
Where do you come across these shapes?

- quadrilateral: is there a "quad"/court in school?
- pentagon: "Pentagon" in US;
- hexagon: they tessellate in bee-hives;
- heptagon: 20p and 50p coins, although they're actually a little rounded at the corners;
- octagon: an octopus has 8 tentacles;
- decagon : "decimal", "decimetre", etc.

triangle" (3) and "square" (4); otherwise we just say "regular" before the name.

The US Pentagon was built in that shape with the idea that it would be quick to get from any part of the building to any other part.

Names for polygons with lots of sides are interesting to some pupils, although we would probably say "46-gon", etc. (see sheet).

2.1.4 Where is there a very large, very well-known triangle?
Where exactly is it?

Answer: (there may be other answers)
The Bermuda Triangle, in which many planes and ships have gone missing over the years. Its vertices are at Bermuda, Miami (Florida) and San Juan (Puerto Rico).

2.1.5 Which letter of the Greek alphabet looks like a triangle?

Answer: Capital delta, the fourth letter of the Greek alphabet, is Δ (the lower case delta is δ), and is used in maths and science, as is the upside down version ∇.

2.1.6 **NEED** square dotty paper, or photocopies of sets of 3 × 3 squares of dots (see sheets).
If every vertex must lie on a dot, how many different triangles can you draw on a 3 × 3 square grid of dots? Count as the same any triangles which are just reflections, rotations or translations of each other?

Answers:
Equilateral triangles are impossible.
See sheet for the others.

You can do a similar task with quadrilaterals (see sheet).

2.1.7 **NEED** acetate of quadrilaterals (see sheet).
"What have all these shapes got in common?" (polygons, 4 sides, quadrilaterals)
"Pick one and tell me what you would call it." "What makes it an X? What does a shape have to have to make it an X?"

You can offer a challenge: "Who thinks they could say the name of every shape?"

This can lead to seeing that all squares are rectangles, rhombuses and parallelograms, etc.

This may be the time to introduce the notation for equal angles, equal sides and parallel sides.

You can turn the acetate by quarter turns and even turn it over to change the appearance and positions of the shapes.

2.1.8 Classifying Quadrilaterals (see sheet).
This is more complicated than it may seem at first sight. You need very careful definitions.

Construct a Venn Diagram or a Flow Diagram for classifying any quadrilateral; e.g., "Are all the sides equal? Y/N", etc.

The table at the bottom of sheet (or one like it) can be drawn on the board and completed by pupils (individually or in groups).

2.1.9 **NEED** photocopies, scissors and glue. Matching definitions (see sheet).
Pupils could work in pairs or individually.
Cut out the statements and the polygon names and match them up. Could stick them down in books if you want a permanent record.

This fits nicely on a double page of a normal exercise book.

2.1.10 Link polygons to co-ordinates (all positive or positive and negative), and kill two birds with one stone.

Pupils can make up their own.

("Plot these points and join them up – name the resulting polygon.")

2.1.11 Choose a volunteer. They stand at the front of the room. Write the name of a polygon on a piece of paper and show it to the pupil. The pupil has to describe it without using the word you've shown them, without drawing anything on the board or waving arms around. When enough information has been given, the pupil chooses another to "guess", and

You can do this in teams or against the clock.

Obviously make sure that the class can't see the word through the paper! (Whispering the word to the pupil is likely to be too insecure!)

You need to decide whether you will allow things like if correct that pupil replaces the one at the front.

"you tie string to it and it flies in the sky"! Really the aim is to be talking about the mathematical properties of the shapes!

You may sometimes need to penalise guessing by deducting points for wrong guesses.

Sometimes you may need to interfere because pupils guess correctly from poor explanations.
"Did it have to be that?" "Have you got enough information yet to rule out every other possibility?"

2.1.12 **NEED** "Finding Quadrilaterals" sheet.
Which kind of quadrilateral isn't there?

Pupils need to use the ABCD labelling convention (clockwise or anticlockwise, start anywhere, don't need to repeat the vertex you start at).

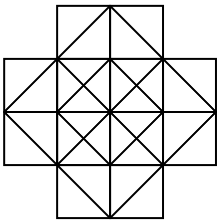
Can also look for different kinds of triangles and for polygons with more than 4 sides.

The drawing is accurate, so pupils can measure lines and angles.
A square is the only one missing.

All the sides of the shape have to be lines that are actually drawn in. If you draw in more lines then there are too many polygons to find.

Pupils can invent their own version, but may need advising not to make it too complicated!

2.1.13 How many squares of any size can you find in this drawing?

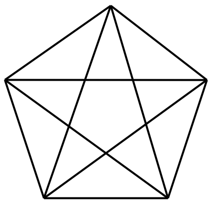


Answer: 27
Be systematic:

- *side length 1, there are 12;*
- *side length 2, there are 5;*
- *side length $\frac{1}{2}\sqrt{2}$, there are 4;*
- *side length $\sqrt{2}$, there are 5;*
- *side length $2\sqrt{2}$, there is 1.*

So the total is 27.

2.1.14 How many triangles of any size can you find in this drawing of a pentagram inside a pentagon?



Answer: 35
Be systematic again:
(edge means edge of the large pentagon)

- *small isosceles, there's 5;*
- *large isosceles, there's 5;*
- *acute-angled containing 1 edge, there's 5;*
- *obtuse-angled containing 1 edge, there's 10;*
- *obtuse-angled containing 2 edges, there's 5;*
- *obtuse-angled inside, there's 5.*

So the total is 35.

2.1.15 Imagination (see sheet).
Pupils may prefer to close their eyes when trying to visualise these.
The teacher can read them out slowly.

Answers: ("Clock Polygons")
1. an equilateral triangle; 2. a square; 3. a regular hexagon; 4. an isosceles triangle; 5. a rectangle; 6. a kite; 7. a scalene triangle; 8. (hard) a right-angled triangle (the angle in a semicircle is always 90°); 9. a trapezium; 10. an irregular pentagon.

Answers: ("Shape Combinations")

1. *an obtuse-angled isosceles triangle OR a parallelogram;*
2. *a different parallelogram OR an arrowhead;*
3. *a rhombus;*
4. *a parallelogram OR a kite;*
5. *a triangle (if you do it to all 8 vertices, the solid you end up with is called a "truncated cube");*
6. *a hexagon (not necessarily a regular one);*
7. *7. a parallelogram OR a concave hexagon.*

2.1.16 Polygon People.
Use pencil and ruler to draw a "polygon person" made entirely out of polygons. Underneath make a table of "body part" and "polygon name".

You can restrict this in some way (e.g., to just triangles). Can make nice display work.
Polygon animals/aliens are obvious alternatives.

Concave polygons are still polygons.

2.1.17 Start with a square piece of paper. With one straight cut, what shapes can you make?

A good task for promoting "exhaustive thinking"

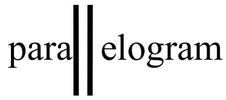
Answer: 2 rectangles (congruent or not); or 2 congruent right-angled isosceles triangles; or 1 right-angled triangle and 1 irregular pentagon containing 3 right-angles; or 1 right-angled triangle *(considering all the possibilities).*

and 1 right-angled trapezium (depending on the angle of the cut and whether it goes through 0, 1 or 2 vertices).

What if you are allowed 2 straight cuts?

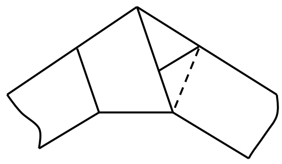
Lots of possibilities now. You can find them all by drawing the 4 possibilities above and considering all the positions of a second line: it could pass through 0, 1 or 2 vertices; if the first line went through a vertex the second one may or may not go through the same vertex; lines parallel and perpendicular to the first line may give different possibilities.

2.1.18 Make a poster of polygon vocabulary illustrating each word to make it easier to remember; e.g., making the double-l in "parallelogram" into a pair of parallel lines.



2.1.19 **NEED** scrap paper, scissors.
Making a Pentagon.
Cut out a thin strip of paper with the same width all the way along.
Tie a very loose knot and flatten it down.
What shape do you expect to get?

Answer: regular pentagon



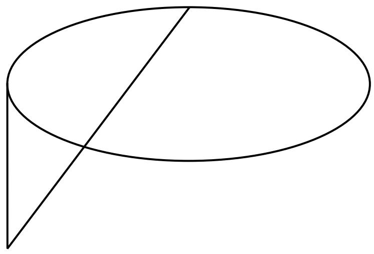
2.1.20 **NEED** Tangrams (bought or made).
You can buy plastic sets of pieces or make your own out of 1 cm × 1 cm A4 squared paper (or A4 card with the shapes photocopied onto it – see sheet).
Many different objects/pictures can be made, ranging from fairly easy to extremely difficult.

All the pieces must be used in each puzzle, and no overlapping is allowed.

It's worth examining the pieces carefully. This happens naturally if pupils make their own.
The most common tangram set (see sheet) comes from cutting up a 4 × 4 square into 7 pieces. There are 2 pairs of congruent right-angled isosceles triangles, another right-angled isosceles triangle, a square and a parallelogram.
The only one worth turning over is the parallelogram (it's the only one without at least 1 line of symmetry).

2.1.21 Describing Designs.
In pairs, pupils turn their chairs so they are sitting back-to-back. One pupil draws a shape or combination of shapes (not too complicated) and the other has some rough paper. The first pupil has to describe orally the shape so that the second can accurately draw it without either pupil seeing the other's paper. The second person isn't allowed to speak. The final shape has to be in the same orientation and about the same size as the original. Team-work is the aim.

e.g., quite a difficult one would be



It's easier to say what to do *rather than what is* there*; e.g., "Put your pen at the centre of the paper and draw a line straight down for about 6 cm" rather than "there's a 6 cm straight line down the middle of the page".*

Polygon Names

number of sides	name	name if regular
1	-	-
2	-	-
3	triangle	equilateral triangle
4	quadrilateral	square
5	pentagon	-
6	hexagon	-
7	heptagon	-
8	octagon	-
9	nonagon (or enneagon)	-
10	decagon	-
11	undecagon (or hendecagon)	-
12	dodecagon	-

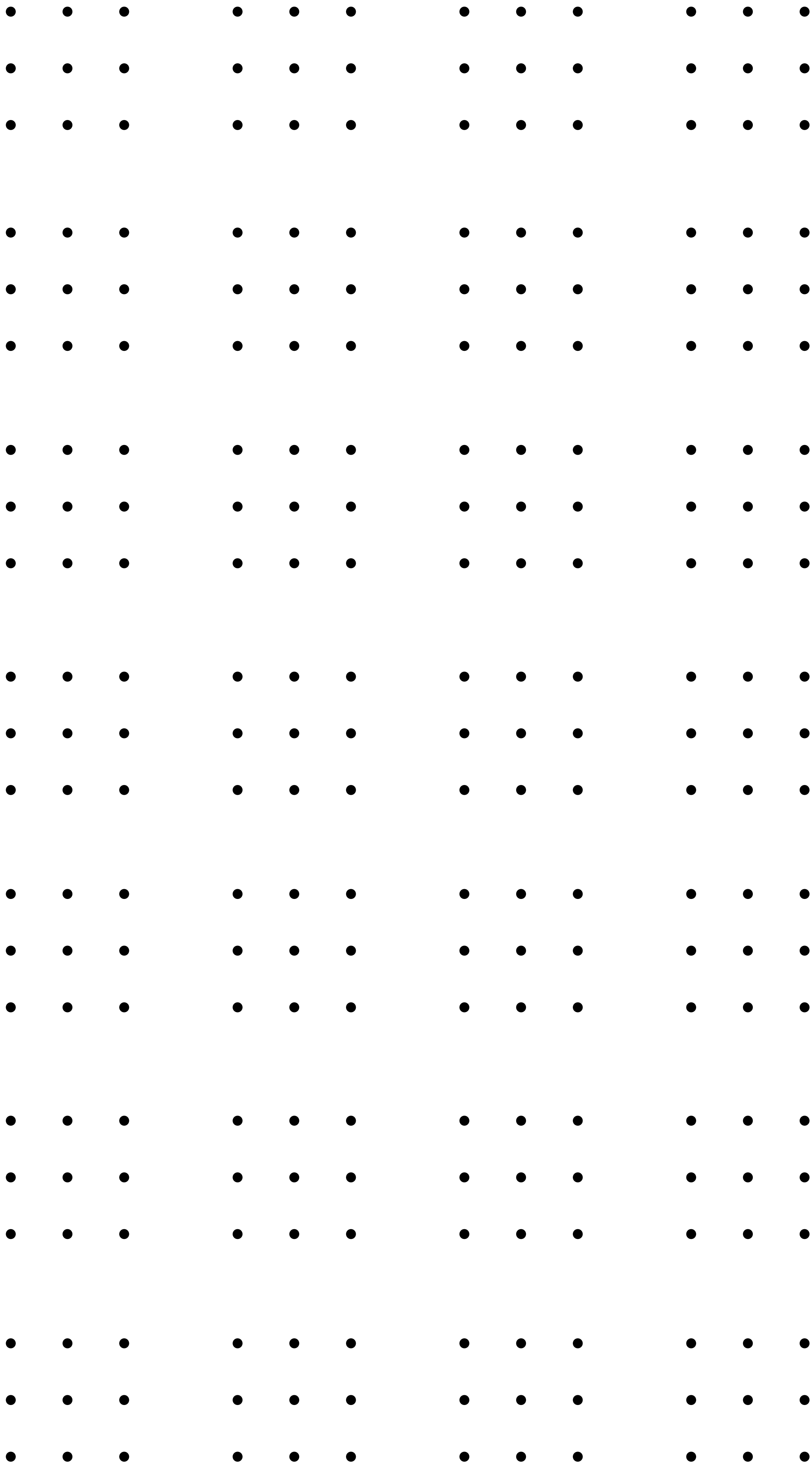
- *Equilateral triangle* (equal sides) and *equiangular triangle* (equal angles) both refer to a regular triangle, whereas a quadrilateral has to be *both* equilateral (rhombus) *and* equiangular (rectangle) to be a square.
- Although other quadrilaterals than squares are common ("regular" in the sense of ordinary), they are not *mathematically regular* because they don't have all their sides of equal length and all their angles the same size.
- A dodecagon has 12 sides; a dodecahedron is a 3-d solid with 12 faces.

Names above 12 are not commonly used, although they are not too complicated.
Some of the higher ones are as follows. Sometimes there is more than one possible name.

number of sides	name
13	tridecagon
14	tetradecagon
15	pentadecagon
16	hexadecagon
17	heptadecagon
18	octadecagon
19	enneadecagon
20	icosagon
30	triacontagon
40	tetracontagon
50	pentacontagon
60	hexacontagon
70	heptacontagon
80	octacontagon
90	enneacontagon
100	hectacontagon (hectogon)
1 000	chiliagon
1 000 000	miliagon

- An icosagon has 20 sides; an icosahedron is a 3-d solid with 20 faces.

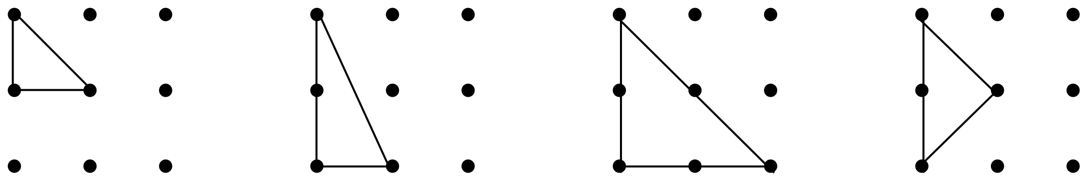
For polygons with lots of sides, you can say, for example, ***46-gon*** for a 46-sided polygon.



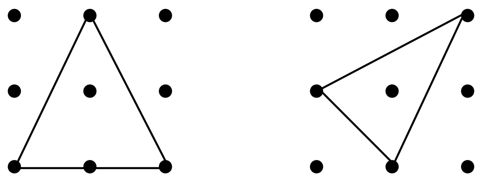
Polygons on 3 × 3 Dotty Grids **ANSWERS**

Triangles

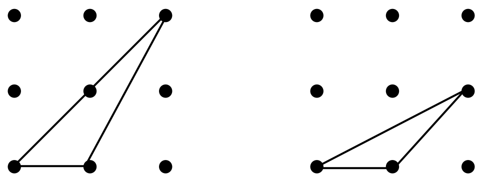
Right-angled



Acute-angled

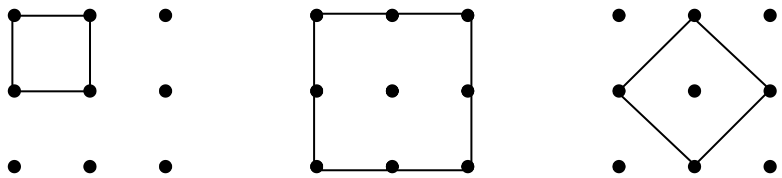


Obtuse-angled

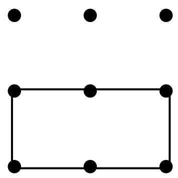


Quadrilaterals

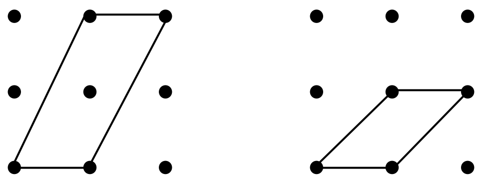
Squares



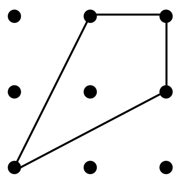
Rectangle



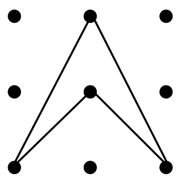
Parallelograms (rhombuses are not possible)



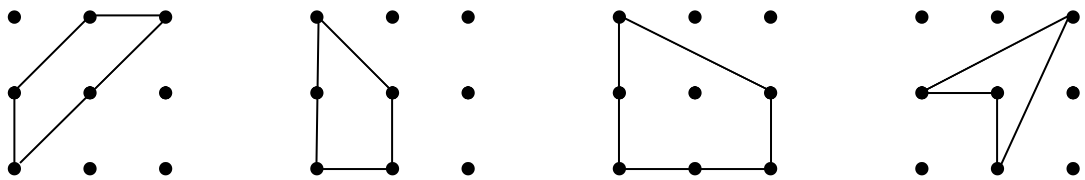
Kite



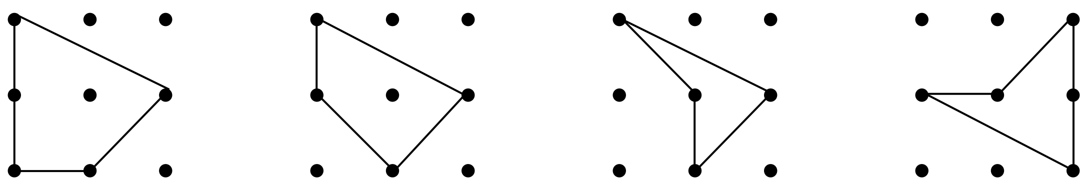
Arrowheads

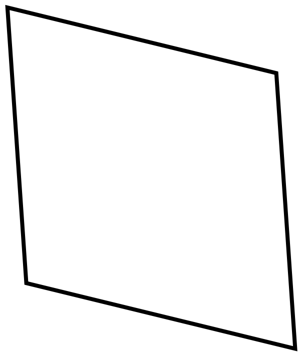
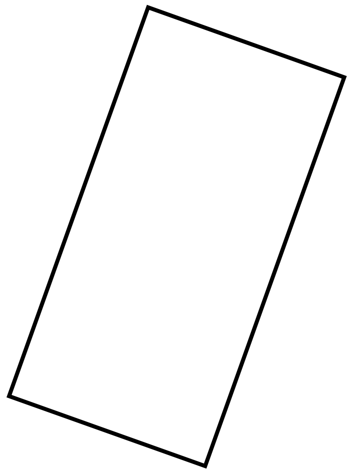
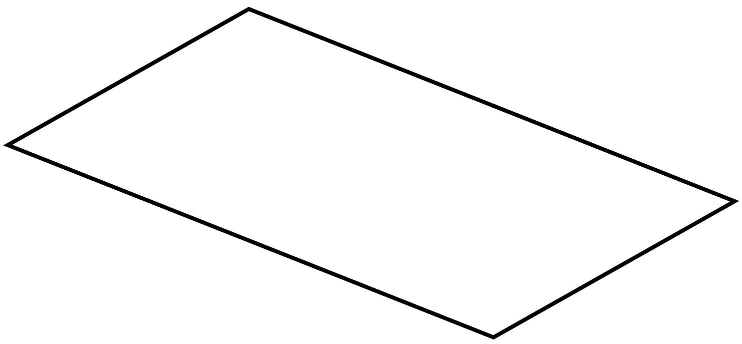
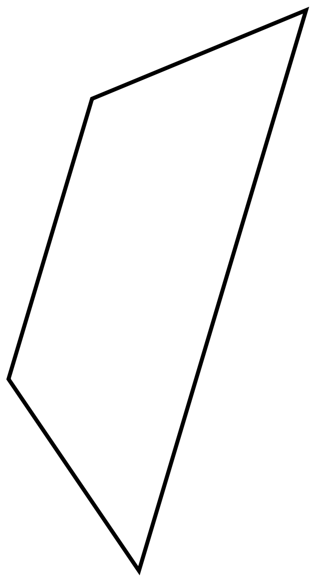
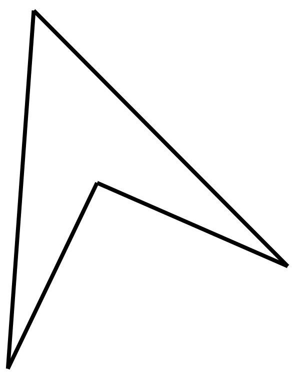
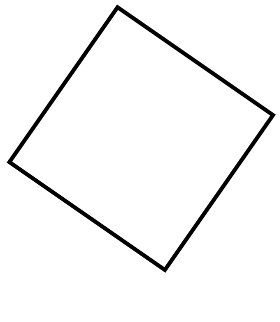
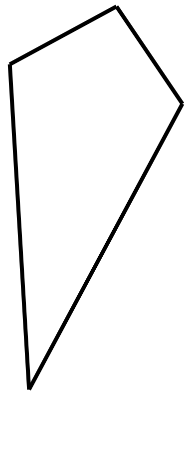


Trapeziums

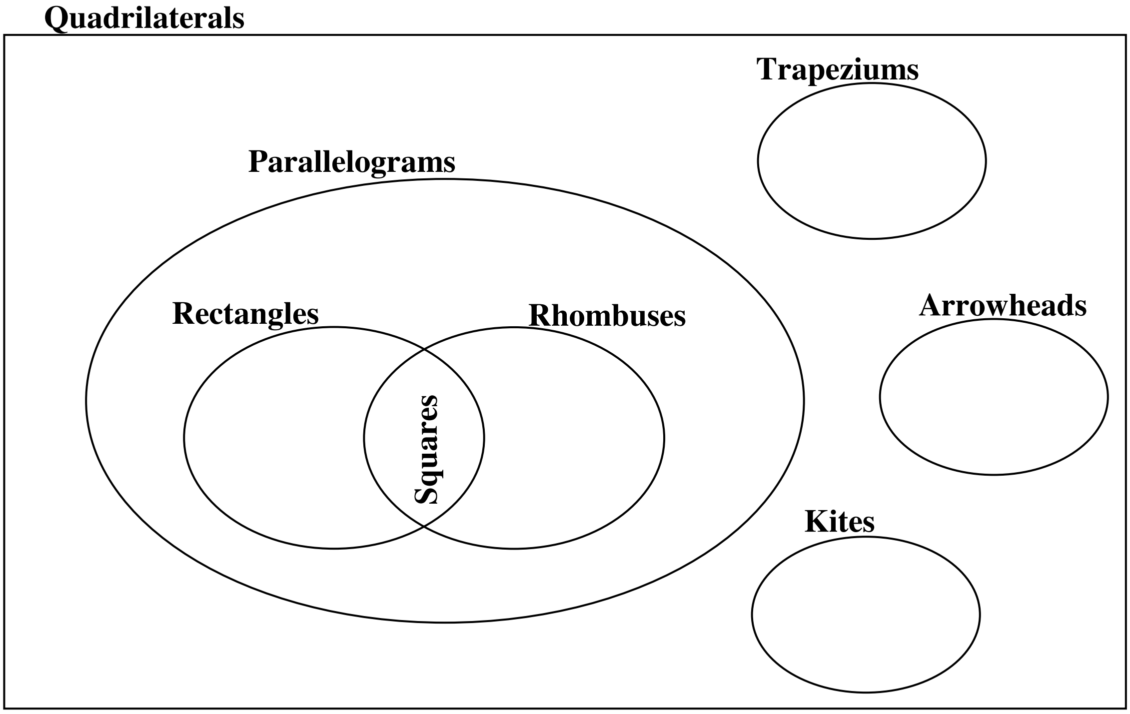


There are also 4 other quadrilaterals (below) that you can draw that don't have special names.



Classifying Quadrilaterals



There are different possible definitions: with more "inclusive" definitions, all parallelograms would count as trapeziums, and kites would include squares, rhombuses and some trapeziums.

Quadrilateral any 4-sided polygon
Parallelogram any quadrilateral with 2 pairs of parallel sides
Rectangle any quadrilateral with 4 right angles
Rhombus any quadrilateral with 4 equal sides
Square any quadrilateral with 4 equal sides and 4 right angles
Trapezium any quadrilateral with only 1 pair of parallel sides
(In an **isosceles trapezium**, the non-parallel pair of sides are of equal length.)
Kite any quadrilateral with 2 pairs of adjacent equal sides (but not all the sides equal) and no interior angle bigger than 180°
Arrowhead any quadrilateral with 2 pairs of adjacent equal sides and one interior angle bigger than 180°

Also, sometimes,
Oblong any rectangle that isn't a square

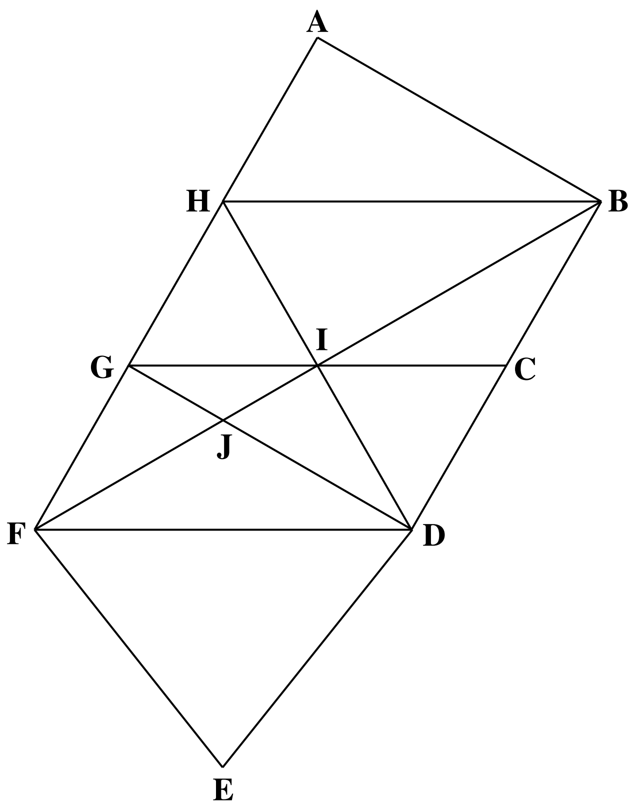
Other properties, such as lines of symmetry, orders of rotational symmetry and properties of diagonals, follow from these definitions.

Properties of Quadrilaterals (things that *must* be so for anything with that name)

	any equal sides?	**any parallel sides?**	**any equal angles?**	**anything else?**
parallelogram	2 opposite pairs	2 opposite pairs	2 opposite pairs	order 2 rot symm
rectangle	2 opposite pairs	2 opposite pairs	all 90°	2 lines of symm
rhombus	all	2 opposite pairs	2 opposite pairs	diagonals at 90°
square	all	2 opposite pairs	all 90°	4 lines of symm
trapezium	1 opposite pair if isosceles	1 opposite pair	2 adjacent pairs if isosceles	1 line of symm if isosceles
kite	2 adjacent pairs	none	1 opposite pair	diagonals at 90°
arrowhead	2 adjacent pairs	none	1 opposite pair	1 reflex angle

I have 4 equal sides and 4 right angles.	hexagon
I have 4 equal sides. The sides are **not** at right angles.	trapezium
I have 4 sides. Opposite sides are equal. Not all the sides are equal. The sides are at right angles.	parallelogram
I have 5 sides.	right-angled triangle
I have 4 sides. Only 2 sides are parallel.	scalene triangle
I have 4 sides. Opposite sides are equal. Not all the sides are equal. There are **no** right angles.	rhombus
I have 6 sides.	kite
I have 3 equal sides and 3 equal angles.	equilateral triangle
I have 3 sides. One corner is a right angle.	rectangle
I have 3 different sides.	isosceles triangle
I have 3 sides. Two sides only are equal.	pentagon
I have 4 sides. Two pairs of sides are equal. Only 1 pair of angles are equal.	square

Finding Quadrilaterals

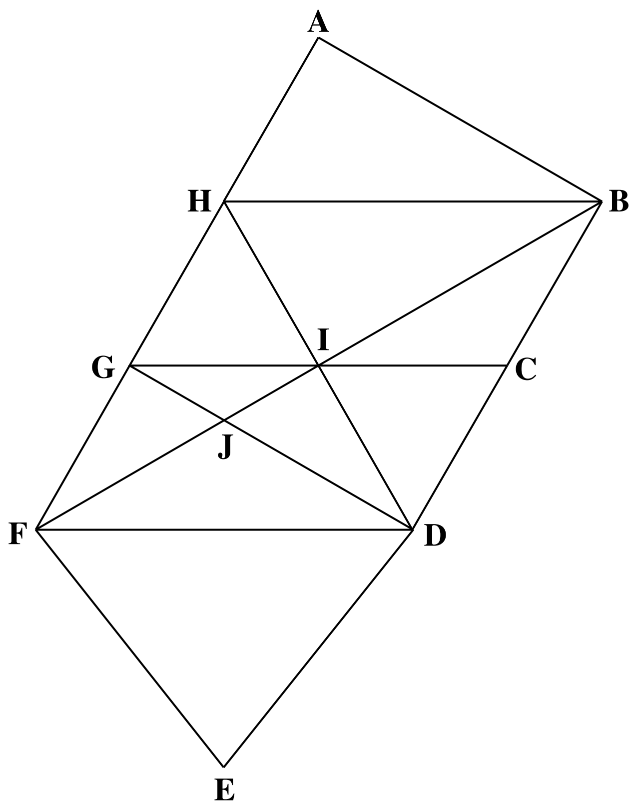


Write down the *quadrilaterals* you can find.

For example, GCDF is a parallelogram

What other polygons can you find?

Finding Quadrilaterals



Write down the *quadrilaterals* you can find.

For example, GCDF is a parallelogram

What other polygons can you find?

Imagination

Clock Polygons

What kinds of polygons do you get if you join up these times?
Ignore the minute hand and the second hand and just think about where the *hour hand* would be.

1 12.00, 4.00, 8.00
2 12.00, 3.00, 6.00, 9.00
3 12.00, 2.00, 4.00, 6.00, 8.00, 10.00
4 1.00, 6.00, 8.00
5 1.00, 3.00, 7.00, 9.00
6 12.00, 5.00, 10.00, 11.00
7 8.00, 12.00, 3.00
8 10.00, 1.00, 4.00
9 10.00, 1.00, 3.00, 6.00
10 9.00, 12.00, 3.00, 5.00, 7.00

Shape Combinations

1 I take a rectangle that isn't a square and cut a straight line along one of its diagonals. I put the two triangles that I get next to each other so that their shorter sides are touching. What are the two possible polygons that I end up with?

2 If I do the same thing with a parallelogram, what are the two possibilities this time?

3 I put two congruent equilateral triangles next to each other so that they touch along one edge. What shape do I get?

4 If I do the same thing with two isosceles triangles, what are the two possible shapes I could end up with?

5 If I cut off one corner of a cube (this is called *truncating* a cube), what flat shape will have been created where the corner was before?

6 If I place two congruent isosceles trapeziums next to each other so that their longest sides are in contact, what shape do I get?

7 If I place two congruent isosceles trapeziums next to each other so that they touch along one of the pair of equal sides, what are the two possible shapes I could end up with?

Tangrams

Cut along all the lines so that you end up with 7 separate pieces.
You have to use all of the pieces for each puzzle.
You are not allowed to overlap any of the pieces.