2.2 Area and Perimeter

- Perimeter is easy to define: it's the distance all the way round the edge of a shape (land sometimes has a "perimeter fence"). (The perimeter of a circle is called its *circumference*.)
  Some pupils will want to mark a dot where they start measuring/counting the perimeter so that they know where to stop. Some may count dots rather than edges and get 1 unit too much.
- Area is a harder concept. "Space" means 3-d to most people, so it may be worth trying to avoid that word: you could say that area is the amount of *surface* a shape covers. (Surface area also applies to 3-d solids.) (Loosely, perimeter is how much ink you'd need to draw round the edge of the shape; area is how much ink you'd need to colour it in.)
- It's good to get pupils measuring accurately-drawn drawings or objects to get a feel for how small an area of 20 cm^2, for example, actually is.
- For comparisons between volume and surface area of solids, see section 2:10.

2.2.1 Draw two rectangles (e.g., 6 × 4 and 8 × 3) on a squared whiteboard (or squared acetate).
"Here are two shapes. What's the same about them and what's different?"
Work out how many squares they cover. (Imagine they're cm^2.) Are there any other rectangles that have an area of 24 cm^2?
Why do you think I chose 24 cm^2 and not 23?
(See related section 2.2.7.)

They're both rectangles, both contain the same number of squares, both have same area.
One is long and thin, different side lengths.

Infinitely many; e.g., 2.4 cm by 10 cm.
23 is prime, so there wouldn't be any all-integer-sided rectangles.

2.2.2 What different units can area be measured in?
When might each be appropriate?
A chart like this may help:

	÷100		÷100		÷100 0	
	⟶		⟶		⟶	
m^2		**are**		**hectare**		**km^2**
	⟵		⟵		⟵	
	×100		×100		×100 0	

Answers: common ones such as cm^2, m^2, km^2, square miles, sq inches, sq ft, etc.

"Are" (metric) should not be confused with the word "area" or the unit "acre" (imperial):
1 acre = 4840 square yards, and
1 acre = 0.4 hectares = 40 ares.

2.2.3 **NEED** A4 1 cm × 1 cm squared paper.
Measure area of closed hand (left if right-handed, right if left-handed) and either foot (remove shoe but not sock).
Count squares which are more than half filled; ignore the others. Put a dot in the middle of squares that you've counted.

See whose foot is closest to exactly 100 cm^2!

Pupils can draw the biggest rectangle (integer sides) possible inside the shape and then use base × height to work out how many squares are there. Then just count the ones round the edge. This saves time.

2.2.4 **NEED** squared or square dotty paper.
Pick's Theorem (1859-1942).
Draw any polygon (not too big or complicated to start with) on the dotty paper. All the vertices must lie on dots.
Work out the area of the polygon.
(Break it up into simpler shapes like triangles or rectangles.)
Count the number of dots inside the shape.
Count the number of dots on the boundary (including the vertices themselves).

Answer:
Let i = number of dots inside the polygon and b = number of dots on the boundary (including the vertices).
Then Pick's Theorem says that $\textbf{area} = i + \frac{1}{2}b - 1$.
Proving this simple-looking formula is hard.

It turns out to have something to do with Euler's formula for polyhedra:
vertices + faces = edges + 2

Look for a connection between these three quantities.

2.2.5 Comparing Area and Perimeter.
It depends what units you use, but if you measure in "units" and "square of the same units", you can ask questions like these.
Draw shapes on 1 cm × 1 cm squared paper which have areas (a , in cm^2) and perimeters (p , cm) connected in the following ways:
The shapes must be made entirely of squares that meet along their edges.

	Connection between p and a	
1	$p = a + 4$	
2	$p = a + 6$	
3	$p = 2a$, a square	
4	$p = 2a$, not a square	
5	$p = a$, a square	
6	$p = a$, not a square	
7	$a = 2p$	

Why are the perimeters of these shapes always even?

If when going round the edge of a shape you move an integer number of spaces to the right, you must come back the same integer number of spaces. Likewise with up and down, so the total number of moves must be even, because it's the sum of two even numbers.

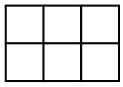
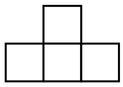
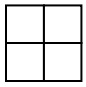
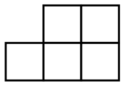
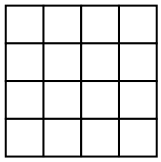
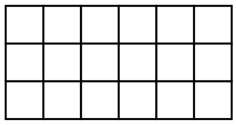
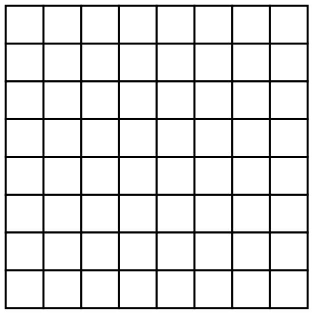
(So this applies only to rectilinear *shapes: polygons where all the interior angles are right-angles.)*

2.2.6 **NEED** pieces of card (see sheet) and OHP.
Area Dissections.
Demonstrate area formulas.

1. Triangle. Start with triangle 1 and label the base as b and the height as h . Introduce the two pieces of triangle 2 (which is congruent to triangle 1) and show that together with triangle 1 they make a rectangle of area bh .
2. Parallelogram. Label the base as b and the height as h . Remove the triangular end and show that it fits onto the other end to make a rectangle of area bh .
3. Trapezium. Label the height as h and the parallel sides as a and b . Introduce the other (congruent) trapezium (upside down) and show that together they make a parallelogram of area $(a+b)h$.

2.2.7 Draw a rectangle with an area of 24 cm^2.

(section 2.9.5).
Answers: (there are many other possibilities)

		a	p
1		6	10
2		4	10
3		4	8
4		5	10
5		16	16
6		18	18
7		64	32

Pupils could use the pieces to prove the formulas to one another.

Implicit in each proof is the idea that you could always perform the same dissections and rearrangements whatever the precise shape.

So triangle area = $\frac{1}{2}bh$.

So parallelogram area = bh .

So trapezium area = $\frac{1}{2}(a+b)h$.
(Notice that if $a = 0$ *(or* $b = 0$ *), the shape becomes a triangle and the area formula becomes* $\frac{1}{2}bh$ *, as it should.)*

Answers:

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(This builds on section 2.2.1.)
Work out its perimeter and write it inside. Repeat. What are the biggest and smallest perimeters you can find?
You are not allowed to change the total area.

Rectangles with integer *sides and area 24 cm²:*

rectangle	perimeter	rectangle	perimeter
1 × 24	*50 (max)*	*2 × 12*	*28*
3 × 8	*22*	*4 × 6*	*20 (min)*

If non-integer values are allowed, then the smallest perimeter would come from the square, which has sides $\sqrt{24} = 4.9$ *units and a perimeter of 19.6 units. The largest would come from a very long thin rectangle* δ *by* $\frac{24}{\delta}$*, where* δ *is small. The perimeter would be* $2(\delta + \frac{24}{\delta})$*, which tends to infinity as* δ *gets smaller and smaller. So you could make the perimeter as large as you like.*

With integer *sides, the smallest perimeter belongs to the rectangle most like a square. The largest perimeter comes from the rectangle of width 1 cm.*

Why do you think I chose 24 cm² and not, say, 23 cm² for the area?

24 has lots of factors; 23 is prime.

What if I fix the *perimeter* at 24 cm, and ask for the biggest and smallest *areas* you can make? Still only rectangles are allowed.

You may need to hint that a square is a rectangle and so is allowed without giving the game away!

Rectangles with integer sides and a perimeter of 24 cm:

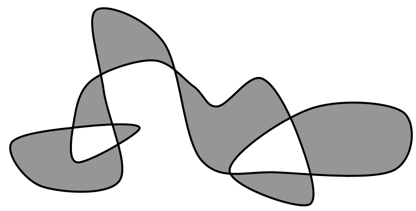
rectangle	area	rectangle	area
1 × 11	*11*	*2 × 10*	*20*
3 × 9	*27*	*4 × 8*	*32*
5 × 7	*35*	*6 × 6*	*36*

Again, the largest area for a given perimeter (or smallest perimeter for a given area) comes from the square.
Areas as small as you like come from rectangles δ *by* $12 - \delta$*, where* δ *is small, and have area* $\delta(12 - \delta)$*, which tends to zero as* δ *gets smaller and smaller.*

(This investigation is extended in section 2.6.9)

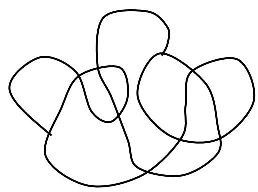
2.2.8 How many colours do you need to colour the countries on a map?
Draw a pattern (not too complicated) without taking your pen off the paper. You can cross over yourself, but you must finish at the point you started.
If you want to colour it in so that always when two areas have a side in common they are different colours, how many colours do you need? (It's OK for the same colours to touch at a point, just not at a side.)

Answer: only 2 colours needed
e.g.,



What if you don't finish at the point you started?

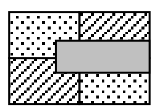
e.g.,



This time you need up to 3 colours.

Can you find a design that needs more than 3 different colours?
(You are allowed to take your pen off the paper during the drawing, now.)

Some designs need 4 colours; e.g.,



(Remember that the outside needs colouring – white in this case.)

Any "map" can be coloured with at most 4 colours. This "Four Colour Theorem" is very hard to prove, but was eventually done using computers. (This was the first major theorem to be proved using a computer.)

2.2.9 Polyominoes (1 cm × 1 cm or 0.5 cm × 0.5 cm squared paper is useful).
Start with dominoes – there's only one possible flat shape you can make by placing two squares next to

Answer:
The numbers grow very quickly.

no. of	name	no. of poly-

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each other. They mustn't overlap and they may only touch edge-to-edge.
What about triominoes (3 squares), and so on? How many different ones can you find?

The number of squares a *is the area, and for* $a \le 3$ *the perimeter* p *is the same for each polyomino of that size and equal to* $2(a+1)$.

The 12 pentominoes are sometimes referred to by the capital letters they look most like: FILNPTUVWXYZ.

squares		*ominoes*
1	*square*	*1*
2	*domino*	*1*
3	*triomino*	*2*
4	*tetromino*	*5*
5	*pentomino*	*12*
6	*hexomino*	*35*
7	*heptomino*	*108*
8	*octomino*	*369*
9	*nonomino*	*1285*

There is no simple pattern to these numbers. (From 7 onwards some of the polyominoes contain holes.)

You can do a similar thing with equilateral triangles (the shapes are called *polyiamonds*) and with hexagons (*polyhexes*).
How many of those can you find?

The three tetriamonds are the three possible nets for a tetrahedron.

no. of triang's	***name***	***no. of poly-iamonds***
1	*equil. triangle*	*1*
2	*diamond*	*1*
3	*triamond*	*1*
4	*tetriamond*	*3*
5	*pentiamond*	*4*
6	*hexiamond*	*12*
7	*heptiamond*	*24*
8	*octiamond*	*66*
9	*enneiamond*	*160*

Polyhexes (made up of regular hexagons) have relevance in organic chemistry because they give the number of possible isomers of some of the aromatic hydrocarbons. For example the three trihexes correspond to anthracene, phenanthrene and phenalene (all $C_{14}H_{10}$*), where the hexagons are rings of carbon atoms with hydrogen atoms attached.*

no. of hex'ns	***name***	***no. of poly-hexes***
1	*hexagon*	*1*
2	*dihex*	*1*
3	*trihex*	*3*
4	*tetrahex*	*7*
5	*pentahex*	*22*
6	*hexahex*	*82*
7	*heptahex*	*333*
8	*octahex*	*1448*
9	*enneiahex*	*6572*

2.2.10 A politician claims that the world isn't overcrowded at all. He says that every person in the world could have an average-sized house (and garden) and the whole lot would fit into California.
What do you think?
What data would you need to test his claim?

Answer: He may be exaggerating, but not that much. Assuming that there are about 6.5×10^9 *people in the world, and each average-sized property measures about 20 m × 20 m, then the total area needed =* $6.5\times10^9\times20\times20 = 2.6\times10^{12}$ *m*^{2}*.*
Since 1 km^2 *=* 10^6 *m*^{2}*, this is only about* 3×10^6 *km*^{2}*.*
The area of the USA is about 9.5×10^6 *km*^{2}*, so this is about* $\frac{1}{4}$ *of that.*

The area of California is about 4×10^5 *km*^{2}*, so he is about 7 times out.*

My aunt says you could fit everyone in the world onto the Isle of Wight if they lined up shoulder-to-shoulder (all standing on the ground). Is that possible?

To make the sums easier, we'll give everyone 0.5 m by 0.5 m = 0.25 m^2*, which should be enough room.*
Total area = $6.5\times10^9\times0.25$ *m*^{2} *=* 1625 *km*^{2}*. This is about 40 km by 40 km, or 635 square miles. This is about four times the area of the Isle of Wight (about 150 square miles).*

Again, my aunt is exaggerating but not all that much.

2.2.11 Display cabinets.
A museum curator wants to arrange her glass display cabinets so that visitors can view the exhibits. She has 9 square cabinets. What is the best arrangement?

Answers:
The cabinets have to touch along the sides.

With a 3 × 3 arrangement, it would be very difficult to see the cabinet that is in the middle, and the others

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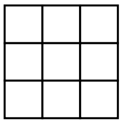
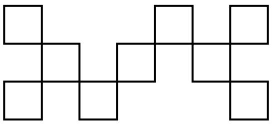


Why would a 3 × 3 square arrangement be a bad idea?

Draw the maximum perimeter arrangement if the cabinets must be connected side to side (not corner to corner).
Find a connection between the number of cabinets and the maximum perimeter.

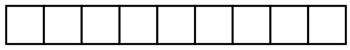
If the cabinets could touch only at the corners, the maximum perimeter would be $4n$; e.g.,



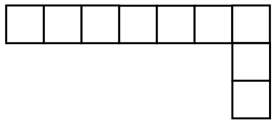
In a different room she wants to use cabinets shaped like equilateral triangles (when viewed from above).
Find a formula for the maximum perimeter when she uses different numbers of these cabinets.
What about other regular polygon cabinets?

could only be viewed from 1 or at most 2 sides.

You want to get the maximum perimeter so that there's the maximum number of sides people can view from.



Always put the cabinets in a single line. In fact simple turns don't affect the perimeter; e.g., this arrangement has the same perimeter (20) as the L-shape below.



If n is the number of cabinets, then the maximum perimeter is $2n+2$.

With n equilateral triangles, the maximum perimeter is $n+2$; for regular pentagons, the maximum perimeter is $3n+2$.
For an r-sided regular polygon, the maximum perimeter is $(r-2)n+2$.

2.2.12 Areas of Parallelograms (see sheet).
Investigating what controls the area of a parallelogram.

Answers (in square units):
A. 10; B. 15; C. 3; D. 6; E. 9; F. 12;
G. 10; H. 10; I. 10; J. 10.

2.2.13 **NEED** A4 1 cm × 1 cm squared paper.
"Design a Zoo" (see sheet).

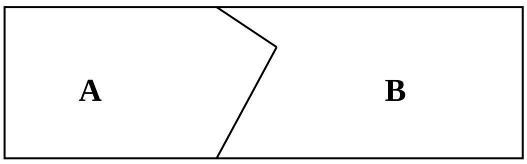
Makes good display work and combines different aspects of maths. An enjoyable task.

2.2.14 A ream of A4 paper is described as 80 g/m^2. What is the mass of an individual sheet?
How much does the whole packet weigh?
How many could I send first class?
(Assume that the envelope weighs 10 g.)

A "ream" of paper is 500 sheets.

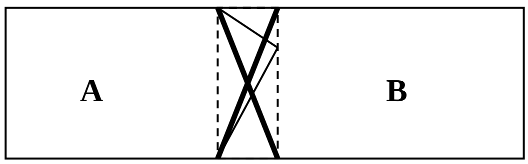
Answer:
1 sheet weighs 0.21 × 0.297 × 80 = 5 g (approx)
500 sheets weigh about 2.5 kg.
If the maximum mass for the cheapest postage rate is, say, 60 g, then we could put 10 sheets of paper in the envelope. (This ignores the mass of the stamp and any ink.)

2.2.15 Two people, Alison and Billy, own some land as shown below.



They want to replace the V-shaped fence with a straight line so that their plots will have a more convenient shape, but they must keep the same amounts of land each.
Where should the line go?
(The land is equally good everywhere.)

Answer:



Draw in the parallel dashed lines as above.
Then the new fence should be either of the thick black lines, because the area of the obtuse-angled triangle is the same as the areas of the right-angled triangles with the same base and the same height, so A can have one of those instead with no change in area.

2.2.16 196 soldiers are marching in a square arrangement. The soldiers at the edge of the square have to carry a flag. How many of these "outside" soldiers are there?
How many "inside" (non-flag-carrying) soldiers are there?

Answers:
For n^2 soldiers, there will be $4n-4$ on the perimeter (4 sides of n soldiers makes $4n$, but the 4 at the corners get counted twice because they each belong to two sides, so we have to subtract those 4).
The inside soldiers make a square arrangement of

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Altogether, $4n-4+(n-2)^2$
$=4n-4+n^2-4n+4$
$=n^2$
as it should do.

$(n-2)^2$.

So for 14^2 (196) soldiers, the number outside is 52 and the number inside is 144, so 52 flags are needed.

2.2.17 **NEED** tape measures, possibly other things as well. Estimate the surface area of a human being.

Practical methods: e.g., wrap someone up in newspaper; use sticky tape and remove the wrapping by cutting carefully with scissors so that when flattened out it approximates the area.

Theoretical methods: e.g., ignore hands, feet, etc., and treat the human body as a sphere on top of a cuboid with two identical cylindrical arms and two bigger identical cylindrical legs. (Different pupils may decide on different assumptions.)

(See similar task in section 2.10.14.)

Answer: the value is not too important – it's the process adopted that matters – but suggested values are given below.

Values will obviously depend on the size of the pupils.

Theoretical approximation:
Head: $4\pi r^2 = 4\pi 10^2$ *= 1300 cm²;*
Trunk:
2 × (50 × 50 + 20 × 50 + 50 × 20) = 9000 cm²;
Arms:
2 × $2\pi rl$ *= 2 × 2 × 3.14 × 4 × 50 = 2500 cm²;*
Legs:
2 × $2\pi rl$ *= 2 × 2 × 3.14 × 6 × 80 = 6000 cm²;*
So total estimate = 18 800 cm² = 2 m² approximately, which seems sensible.
(Lungs have surface area of about 100 m², and the intestines about 300 m²!)

2.2.18 Estimate how many tins of paint you would need to paint this classroom.

Before you start decide if there's anything you need to ask me?

Again, the thinking that pupils go through is much more important than the final estimate.

e.g., how many coats of paint?; are we painting behind the cupboards?; are we doing the ceiling? are we doing the door? etc.

You could state that an "average" tin of paint will cover about 15 m².

2.2.19 Heron's Formula (Heron of Alexandria, about AD 10-75) (see sheet).
A formula for calculating the area of a triangle given only the lengths of the sides.

Also known as Hero's Formula.

This formula probably should be more widely known and used.

2.2.20 Tolstoy (1828-1910) wrote a short story called "How Much Land Does A Man Need?" in which a peasant man called Pahóm is offered some land at a price of "1000 roubles *per day*".
It turns out that he can have as much land as he can go round by foot between sunrise and sunset, but he must finish back where he started before the sun goes down.
What would your strategy be if you wanted to get as much land as possible?

(It's an interesting story pupils may like to read – not too long and with a twist at the end – and impressive to say you've read some Tolstoy!)

What shape path would you take? Would you run or walk? Would it be more efficient in the long run to take breaks? What if you saw some particularly good land? What would you do about hills?

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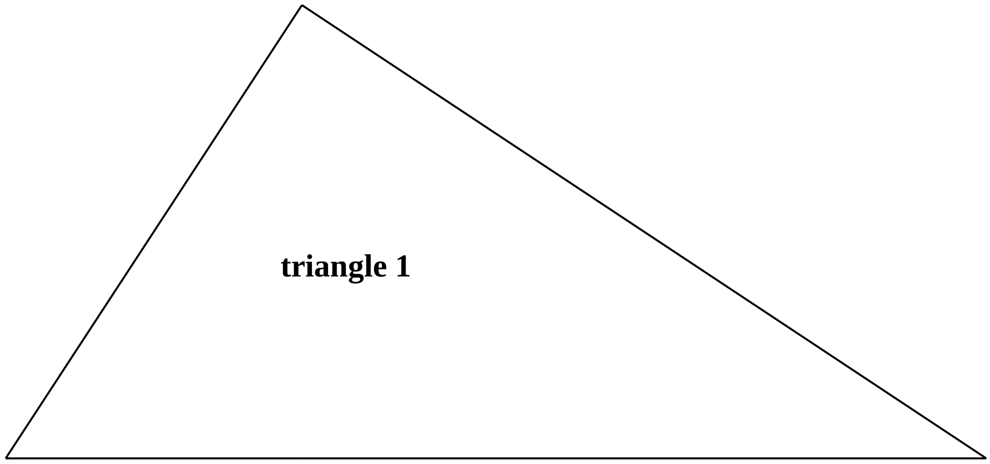
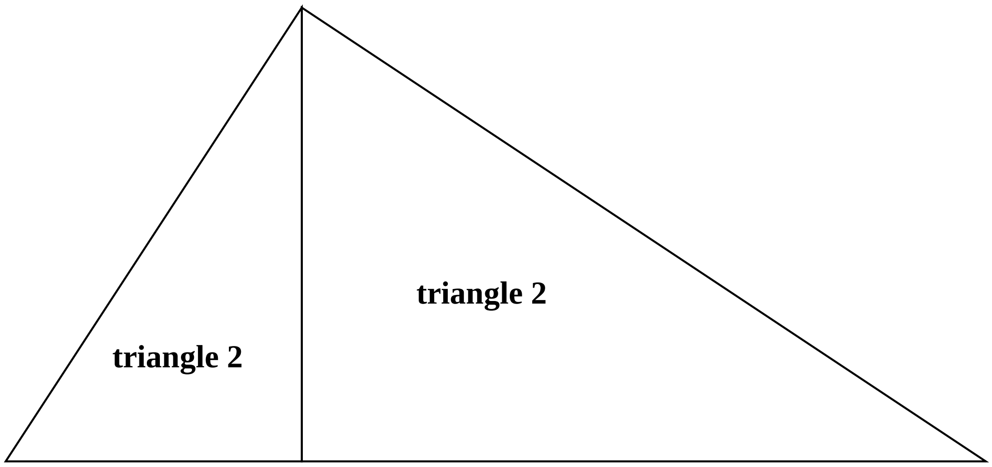
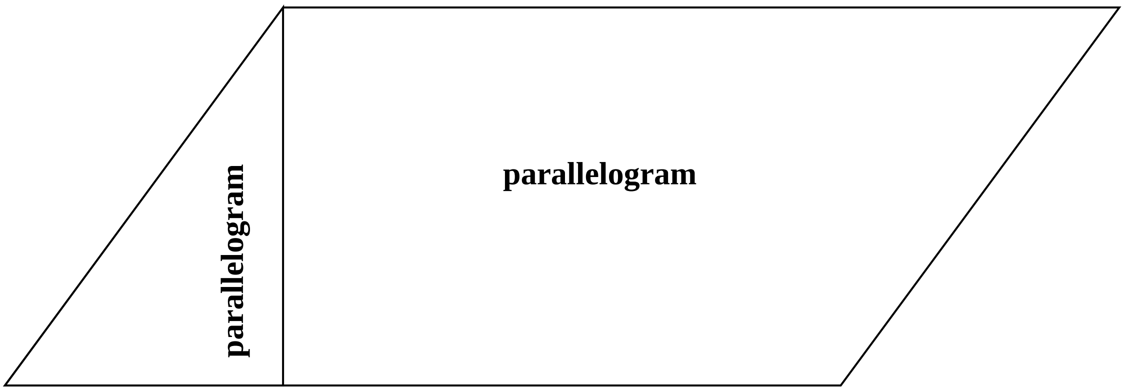
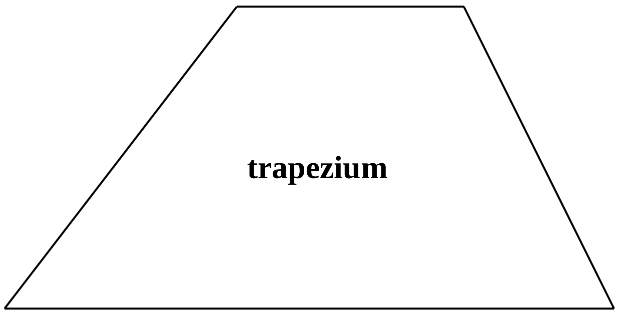
Area Dissections

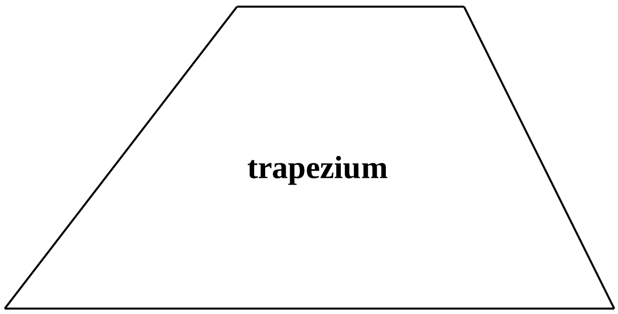
Photocopy onto card and cut along *all* the lines. Keep the pieces in an envelope.
Use to demonstrate area formulas (see notes).











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Areas of Parallelograms

Draw some axes from 0 to 8 horizontally and vertically.
One set should do (with a bit of overlapping) for A to F, and another one for G to J.

Plot each of these parallelograms.
Work out their areas by breaking them into triangles or rectangles.
Record your results.

A	(1, 6)	(2, 8)	(7, 8)	(6, 6)
B	(0, 6)	(5, 6)	(7, 3)	(2, 3)
C	(7, 5)	(8, 6)	(8, 3)	(7, 2)
D	(0, 2)	(0, 4)	(3, 2)	(3, 0)
E	(4, 0)	(3, 3)	(6, 3)	(7, 0)
F	(1, 5)	(3, 7)	(7, 5)	(5, 3)
G	(0, 6)	(1, 8)	(6, 8)	(5, 6)
H	(0, 4)	(2, 6)	(7, 6)	(5, 4)
I	(0, 2)	(3, 4)	(8, 4)	(5, 2)
J	(1, 0)	(0, 2)	(5, 2)	(6, 0)

What things affect the area of a parallelogram and what things make no difference?

Areas of Parallelograms

Draw some axes from 0 to 8 horizontally and vertically.
One set should do (with a bit of overlapping) for A to F, and another one for G to J.

Plot each of these parallelograms.
Work out their areas by breaking them into triangles or rectangles.
Record your results.

A	(1, 6)	(2, 8)	(7, 8)	(6, 6)
B	(0, 6)	(5, 6)	(7, 3)	(2, 3)
C	(7, 5)	(8, 6)	(8, 3)	(7, 2)
D	(0, 2)	(0, 4)	(3, 2)	(3, 0)
E	(4, 0)	(3, 3)	(6, 3)	(7, 0)
F	(1, 5)	(3, 7)	(7, 5)	(5, 3)
G	(0, 6)	(1, 8)	(6, 8)	(5, 6)
H	(0, 4)	(2, 6)	(7, 6)	(5, 4)
I	(0, 2)	(3, 4)	(8, 4)	(5, 2)
J	(1, 0)	(0, 2)	(5, 2)	(6, 0)

What things affect the area of a parallelogram and what things make no difference?

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Design a Zoo!

(Teachers' Notes)

- There will be **10 animals** to house in the zoo.

- Each animal will have a separate **cage**.
  All the cages will be made of 6 squares but arranged differently so that each cage is a different shape.

- Take an A4 piece of 1 cm × 1 cm squared paper.
  This will be your plan for the zoo.
  Use a scale of 1 cm to 1 m and draw the cages, spreading them out over the page.
  Each cage will have an area of 6 m^2.
  Label which cage is for which animal.

- The floor material for the cage will cost £500 per m^2. (All prices include labour!)
  So for 10 cages you will have to spend 10 × 6 × 500 = £30 000.
  Head another sheet of paper "Accounts" and record this cost.
  Show how you worked it out.

- Every cage needs **fencing** round the edge.
  This costs £200 per m.
  Calculate the perimeter of each cage – they won't all be the same.
  Find the total of all the perimeters and multiply this by £200 to find the total fencing cost.
  Put all of this on your accounts sheet.

- Your **budget** for designing the whole zoo is **£250 000**.
  You cannot go over this.

- **Things to add:**
  - **path** to take visitors round the zoo so they can look into each cage.
    Design it and work out how much it will cost.
    Cost = £100 per m^2.
  - **signs** to show the visitors what's where
    Cost = £50 each
  - **car park**
    Cost = £50 per m^2 of gravel
  - **trees**
    Cost = £25 each
  - **toilets**
    Cost = £7 500
  - **café**
    Cost = £25 000

- What other things could you add? Your teacher will give you a quotation!
  You must keep within budget!
  How much would you charge people to visit the zoo?

- Do you think that the cages would be large enough?
  Do you think that the prices are realistic?

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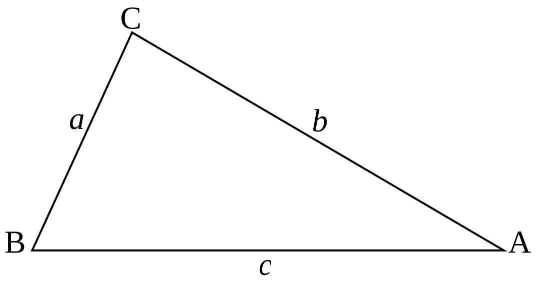
Heron's Formula

Consider any triangle in which the lengths of the sides a, b and c are known and we wish to find the area.



We can use the cosine rule to work out one of the angles (C) and then use the formula **area** $= \frac{1}{2}ab\sin C$ to find the area.

Using the cosine rule, $c^2 = a^2 + b^2 - 2ab\cos C$, so $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$.

Using the identity $\sin^2 C + \cos^2 C \equiv 1$, we can find an expression for $\sin C$, and we get

$$\sin C = \sqrt{1-\cos^2 C} = \sqrt{1-\left(\frac{a^2+b^2-c^2}{2ab}\right)^2} = \sqrt{\frac{4a^2b^2-(a^2+b^2-c^2)^2}{4a^2b^2}}.$$

Now using **area** $= \frac{1}{2}ab\sin C$ we get **area** $= \frac{1}{2}ab\sqrt{\dfrac{4a^2b^2-(a^2+b^2-c^2)^2}{4a^2b^2}} = \frac{1}{4}\sqrt{4a^2b^2-(a^2+b^2-c^2)^2}$.

Factorising the difference of two squares inside the square root sign gives

area $= \frac{1}{4}\sqrt{\{2ab+(a^2+b^2-c^2)\}\{2ab-(a^2+b^2-c^2)\}}$, and rearranging and factorising again gives

area $= \frac{1}{4}\sqrt{\{(a+b)^2-c^2\}\{c^2-(a-b)^2\}}$.

Again using the difference of two squares we get

$$\begin{aligned}
\textbf{area} &= \tfrac{1}{4}\sqrt{(a+b+c)(a+b-c)(a-b+c)(c-a+b)} \\
&= \sqrt{\left(\frac{a+b+c}{2}\right)\left(\frac{a+b-c}{2}\right)\left(\frac{a+c-b}{2}\right)\left(\frac{b+c-a}{2}\right)} \\
&= \sqrt{\left(\frac{a+b+c}{2}\right)\left(\frac{b+c-a}{2}\right)\left(\frac{a+c-b}{2}\right)\left(\frac{a+b-c}{2}\right)} \\
&= \sqrt{s(s-a)(s-b)(s-c)}
\end{aligned}$$

where $s =$ the semi-perimeter $= \dfrac{a+b+c}{2}$.

This formula **area** $= \sqrt{s(s-a)(s-b)(s-c)}$ for the area in terms of the semi-perimeter s and the sides a, b and c is called **Heron's Formula**.

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