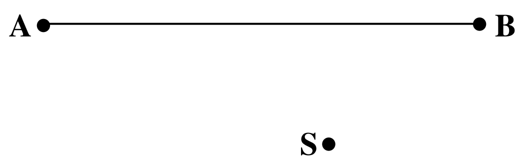
2.5 Bearings

- There are three rules of bearings: always measure from north;
  always measure clockwise;
  always give 3 digits.
  (If there's someone in the class who does orienteering, you could ask him/her (in advance) to bring a compass and teach the class how bearings work.)
  Sometimes you have to draw in a north line at the point you're measuring the bearing *from*.
- A full-circle protractor (0°-360°) is a lot more convenient than a semicircle (0°-180°) one, and it's an advantage if there is no area missing in the middle and there are continuous lines going out from the centre to the numbers around the edge. Pupils also sometimes find a 360˚ protractor easier to hold because of the lump at the centre. The reason for the two scales (clockwise and anticlockwise) may need explaining.
- Pupils can aim for an accuracy of ± 1° and ± 1 mm. A sharp pencil helps. Sometimes lines need extending on a drawing to reach the fine scale around the edge of the protractor.
- Local maps are usually much more interesting than remote or invented ones in textbooks.
  (You could ask the Geography department if you can borrow some local maps.)

2.5.1 **NEED** A4 plain paper.
Coastguard Stations.
Facing the class, pick a pupil at your front left. "Beth is a coastguard station. Ashley (front right) is another coastguard station. And Sally (somewhere in the middle of the room) is a sinking ship!"
On the board, begin a drawing (see right).

Fits nicely on A4 paper.

(The drawing below is the right way round for the class.)



Sally radios in to Ashley and to Beth to say that she's sinking – send help! But her GPS is broken and she doesn't know where she is. And by the way it's night time and foggy.

A and B measure the directions they receive Sally's message from.

	from A	**from B**
Ship	157°	213°

(GPS = Global Positioning System – a very accurate way of telling where you are that uses satellites.)

The coastguard stations are 16 km apart, so draw a line at the top of the page (portrait orientation) 16 cm long (the scale is 1 cm to 1 km). We'll take north as up the page.
Do a scale drawing to find out how far away S is from A and B.

This is called triangulation.

A helicopter radios in to go and rescue S.

	from A	**from B**
Helicopter	103°	244°

Find where the helicopter is and work out how far the helicopter has to go and on what bearing to pick up Sally.

Answers:

	from A	**from B**
Ship	16.2 km	17.8 km
Helicopter	11.1 km	5.7 km

Message to helicopter: "Travel 13.2 km on a bearing of 200° to intercept the ship."

(Although this scenario is not completely realistic, pupils generally realise that but enjoy it anyway.)

(All of this assumes that the ship is stationary.)

2.5.2 **NEED** "Crack the code" sheets, protractors.

Answer:
The message is "Always give three digits!"
The key is as follows.

Pupils can make up their own messages for each other using the same code.

("Remember I know the code so don't say anything you wouldn't want me to read!")

A	*243*	**B**	*016*	**C**	*260*	**D**	*314*
E	*148*	**F**	*234*	**G**	*306*	**H**	*072*
I	*209*	**J**	*167*	**K**	*096*	**L**	*056*
M	*036*	**N**	*124*	**O**	*105*	**P**	*278*
Q	*224*	**R**	*029*	**S**	*183*	**T**	*332*
U	*251*	**V**	*291*	**W**	*346*	**X**	*196*
Y	*269*	**Z**	*133*				

2.5.3 What is the connection between the bearing of A from B and the bearing of B from A?
Can you write a rule that works for any bearing ≥ 000° and < 360°?

Answer:
If the bearing of A from B is θ *, then the bearing of B from A is*
$\theta+180$ *if* $0 \le \theta < 180$ *, and*
$\theta-180$ *if* $180 \le \theta < 360$.
Or you can say $(\theta+180) \bmod 360$.

2.5.4 How could this happen?
I take off in my aeroplane and head South. I turn 90° to the left and head East. After a while, I turn 90° to the left again and head North, and without changing direction again, I land back where I started.

Euclid (about 330-270 BC) wrote "Elements".

The sum of the angles in a triangle on a sphere (a "spherical triangle") comes to more than 180°, more the larger the triangle. Very small triangles behave more or less Euclidean.
What kind of surface would have the sum of the angles in a triangle <180°?

Triangles behave "normally" on the surface of a cylinder. You can unroll a surface like this into a flat sheet; it has no "intrinsic curvature".

Answer: I must have started and finished at the North pole; the middle leg of the journey was along the equator.

A globe or someone's football makes this clear. When you think about angles on a sphere, you're doing non-Euclidean geometry (although all the North lines are going in the same direction, they meet – at the North pole).

(People sometimes think that the answer is the inside *of a sphere, because it curves* away *from you instead of* towards *you, but imagine that the sphere was made of glass – it's actually the same triangle on both sides.)*
The answer is to do with hyperbolic geometry – you need a surface with "negative curvature"; e.g., a bicycle saddle.

2.5.5 **NEED** maps of the local area; an A4 portion should be enough (you could ask the Geography department if there are some you could borrow).
Find the school, draw in a North line there, and work out the bearings off all the major places from the school.
Record the values in a table (place in one column, bearing in another).

Depending on how wide your catchment area is, you may want about 3.5 inches to a mile or 1.25 inches to a mile so as to include most of the places where pupils live or go.

2.5.6 Treasure Map.
Make one, perhaps for an island, mark on the starting point and mark faintly in pencil an X where the treasure is. Draw on dangers (swamps, man-eating tigers, sharks, dangerous rocks, cliffs, etc.), but make sure there's a safe route from the starting point to the treasure. Decide which direction is North, mark that on, and put a scale (say 1 cm = 1 m). On a separate sheet list instructions using bearings for getting safely to the treasure; e.g., "Go 5 m on a bearing of 045°, then go …".
Rub out the X thoroughly afterwards!

If pupils are likely to spend a long time drawing the island and its hazards and not much time working out bearings, you could insist that they mark the safe route as they draw in the dangers and work out the bearings and instructions as they go along, rather than leaving that until the end!

Pupils could do the route on a second (thin) piece of paper laid on top and remove it at the end.

See if someone else can follow your route.

2.5.7 What methods are there for finding where North is?
What are the advantages and disadvantages of each?

Answers: (some possibilities)
- *compass (simple, but only works if you have one handy and aren't too near anything with a strong magnetic field);*
- *use a map (or local knowledge) and a landmark;*
- *point the minute hand of an analogue watch at the sun, and bisect the angle between it and the hour hand (don't need special equipment, but*

only works if you can see the sun);
- *find the North Star (need to know how to locate it, have to be able to see the stars);*
- *global positioning system (very accurate, but obviously you need to have one, and this was no good before they were invented!).*

2.5.8 **NEED** "Fancy Fields" sheets.
An opportunity to review polygon names and scale drawing.

Answers:

A	*rectangle*	***B***	*equil. triangle*
C	*regular hexagon*	***D***	*square*
E	*isos. r-angled tri*	***F***	*isos. trapezium*
G	*parallelogram*	***H***	*regular octagon*

2.5.9 Plot these points and work out the bearings of the *first* from the *second*.
Take the y -axis as North.

1. (2,3) from (0,0)
2. (4,4) from (0,0)
3. (–2,5) from (0,0)
4. (6,–1) from (0,0)
5. (–2,–1) from (0,0)
6. (3,0) from (1,1)
7. (3,1) from (5,2)
8. (–4,–1) from (0,1)
9. (–5,3) from (–2,0)
10. (3,–5) from (–2,–1)

Why are the answers to three of the questions the same?

This could be done by measuring accurately or by calculating using trigonometry and angle facts.

Answers:

1	*034°*	***2***	*045°*
3	*338°*	***4***	*099°*
5	*243°*	***6***	*117°*
7	*243°*	***8***	*243°*
9	*315°*	***10***	*129°*

The lines in questions 5, 7 and 8 all have the same gradient (0.5) and direction (they're the same vectors*).*

2.5.10 Which way does an easterly wind go?

On a map, "northings" run east-west and "eastings" run north-south.

Answer: An "easterly wind" usually means **from** *the east; i.e., heading west, whereas an "easterly current" goes* **to** *the east (from the west).*

For this reason, it's generally clearer to describe the direction it's coming from or going to explicitly.

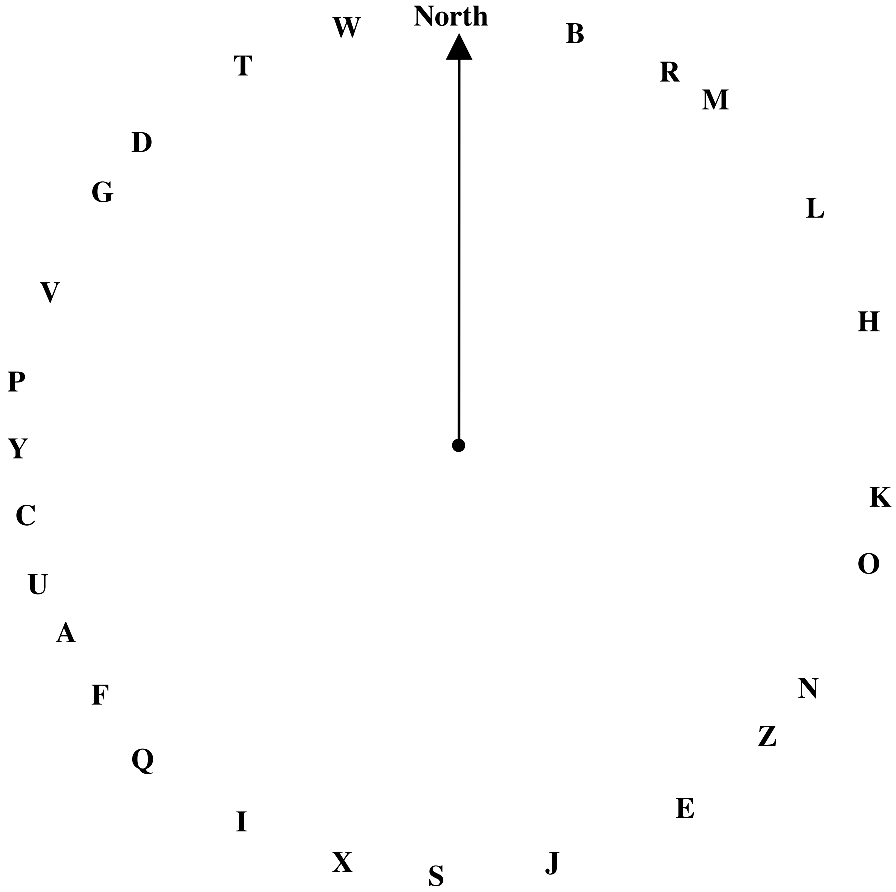
2.5.11 In some religions, people face in a particular direction to pray. Find out more about this.

This would theoretically be impossible if you were either at Jerusalem/Mecca or on the exact opposite side of the world from there (on a ship in the middle of the Pacific Ocean!).

Answer: Some Jewish, Christian and Muslim groups pray facing East. This may have had something to do with the sun rising in the East, or the Garden of Eden being planted "in the East" (Genesis 2:8) or with many believers living to the west of the "Holy Land". Some Jews face Jerusalem from wherever they are along the direction of a Great Circle, and Muslims face Mecca by a similar method.

Crack the Code (Bearings)

Follow the bearings to find the letters.
What is the message?



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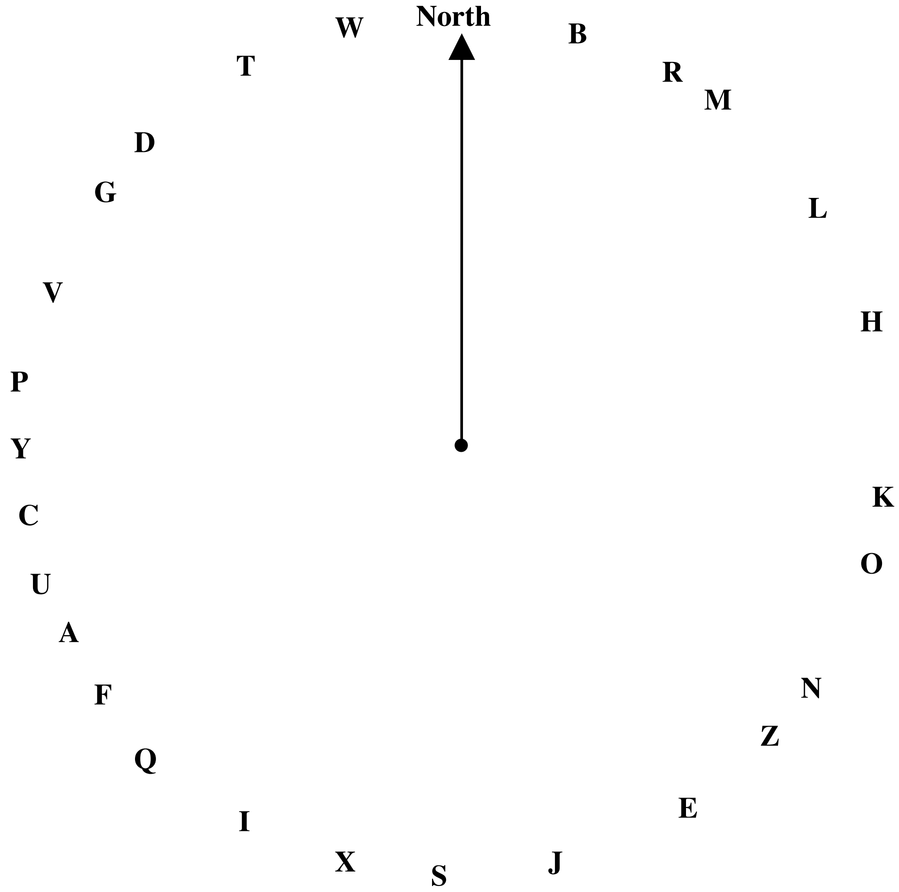

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Fancy Fields

I have eight interesting-shaped fields on my land.
I have walked around the perimeter of each of them recording the bearings and distances for each of the sides.
Look at my data and try to say what shape each field is.
Then do an accurate drawing for each one to see if you are right.
Use a scale of 1 cm representing 100 m.

Field A
A bearing of 000° for 400 m;
then a bearing of 090° for 800 m;
then a bearing of 180° for 400 m;
and finally a bearing of 270° for 800 m.

Field B
A bearing of 000° for 400 m;
then a bearing of 120° for 400 m;
and finally a bearing of 240° for 400 m.

Field C
A bearing of 090° for 500 m;
then a bearing of 150° for 500 m;
then a bearing of 210° for 500 m;
then a bearing of 270° for 500 m;
then a bearing of 330° for 500 m;
and finally a bearing of 030° for 500 m.

Field D
A bearing of 045° for 600 m;
then a bearing of 135° for 600 m;
then a bearing of 225° for 600 m;
and finally a bearing of 315° for 600 m.

Field E
A bearing of 000° for 400 m;
then a bearing of 135° for 566 m;
and finally a bearing of 270° for 400 m.

Field F
A bearing of 037° for 500 m;
then a bearing of 090° for 500 m;
then a bearing of 143° for 500 m;
and finally a bearing of 270° for 1100 m.

Field G
A bearing of 090° for 600 m;
then a bearing of 220° for 400 m;
then a bearing of 270° for 600 m;
and finally a bearing of 040° for 400 m.

Field H
A bearing of 135° for 400 m;
then a bearing of 090° for 400 m;
then a bearing of 045° for 400 m;
then a bearing of 000° for 400 m;
then a bearing of 315° for 400 m;
then a bearing of 270° for 400 m;
then a bearing of 225° for 400 m;
and finally a bearing of 180° for 400 m.

Extra Task Design some fields of your own and work out instructions (using bearings and distance) for placing the fence around the edge.