2.14 Tessellations

- A misconception here is that the interior angles in shapes that tessellate have something to do with the *factors* of 360. That cannot be right, because 360 is an arbitrary number that we choose to divide a whole turn into. We could have used 100, or 2π (could mention radians here), or 359, which is prime.
  See sheet for an explanation of why only some polygons tessellate.

2.14.1 Polyominoes.
Obviously dominoes will tessellate because they are just rectangles.
What about both of the triominoes?
What about the tetrominoes?
And so on …

Answers:
All triominoes, tetrominoes, pentominoes and hexominoes tessellate.
From heptominoes onwards they don't all tessellate.

See section 2.2.9 for how many of each of the polyominoes there are.

2.14.2 What kinds of triangles tessellate?

Answer: all triangles tessellate, because any two congruent triangles will make a parallelogram if you put a pair of corresponding sides together, and parallelograms tessellate.

2.14.3 What kinds of quadrilaterals tessellate?

Answer: again all will, even concave ones.

2.14.4 Design a tessellating shape. You can "force" it to tessellate by starting with something that certainly tessellates (e.g., a parallelogram) and doing opposite things to opposite sides (e.g., cut out a triangle from one side and add it on to the opposite parallel side).

This can make good display work.

Christmas trees are possible.

2.14.5 **NEED** cardboard or plastic polygons.
(You could use the polygon shapes from section 2.1.)
Draw round them and see which ones you can get to tessellate.
To start with, try only one type of regular polygon in each pattern. You should find 3 "regular tessellations". (See sheet.)

It really is worth using cardboard (the thicker the better) and not paper to make templates, because they are much easier to draw round. You can get a lot out of one A4 sheet of card.

2.14.6 **NEED** Escher (1898-1972) drawings (see books).

Remarkable examples of intricate tessellations.

2.14.7 Where have you seen beautiful tessellations?

Answers (suggestions):
Islamic art, mosaics (are there some in school or could people bring in photos?).

2.14.8 Where are tessellations not just pretty but useful?

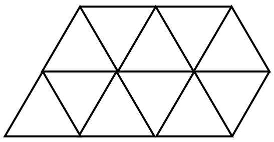
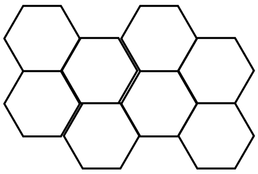
Answers (suggestions):
1. *bee-hive: very sensible structure (rigid);*
2. *some molecular structures (e.g., graphite is made up of sheets of tessellating hexagons of carbon atoms);*

Answers (continued):
3. *squared/isometric paper;*
4. *rigid bridge structures (equilateral triangles);*
5. *kitchen tiles (no gaps is important, because water would get through).*
6. *paving slabs, brick walls.*

Tessellations

A tessellation is a pattern of shapes which cover all of the surface with no gaps and no overlapping.

There are 3 Regular Tessellations – all the shapes are the same regular polygon and all the vertices are the same.

Equilateral Triangles	**Squares**	**Hexagons**
		

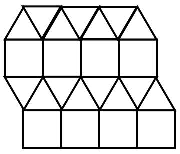
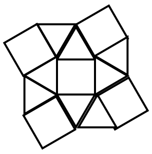
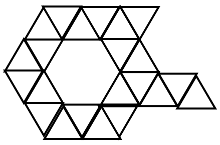
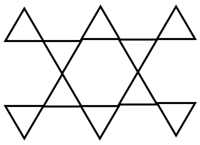
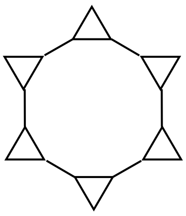
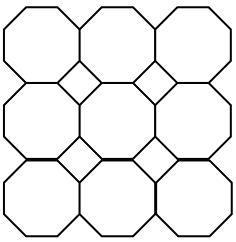
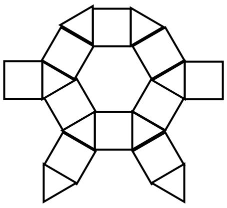
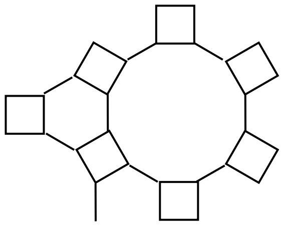
In a regular polygon with n sides, each interior angle is $\frac{180(n-2)}{n}$.

If m (a positive integer) of them meet at a point, then $\frac{180(n-2)m}{n} = 360$, and this simplifies to

$\frac{m(n-2)}{n} = 2$, or $m = \frac{2n}{n-2}$. So if $\frac{2n}{n-2}$ is an integer, the regular polygon will tessellate.

This happens only when n = 3, 4 or 6 (equilateral triangles, squares and hexagons).

There are 8 Semiregular Tessellations – all the shapes are regular polygons, but they're not all the *same* regular polygon. All the vertices are still the same.

3,3,3,4,4	**3,3,4,3,4**	**3,3,3,3,6**	**3,6,3,6**
			
3,12,12	**4,8,8**	**3,4,6,4**	**4,6,12**
			

Every vertex has the same arrangement of regular polygons around it.
Going clockwise or anticlockwise around a vertex, the number of sides on each of the polygons present make the sequences of numbers above (e.g., 3,3,3,4,4 means that at each vertex you have triangle-triangle-triangle-square-square).