THE NTH. TERM

Colin Foster

I very much dislike setting pupils exercises to do in which the answers are of no consequence. For instance, it seems to me (and often to them) a waste of effort to substitute a relatively random list of numbers into a relatively random list of formulæ, simply in order to practise substituting numbers into formulæ. As soon as an answer is obtained, the pupil is encouraged to go straight on to the next question without a thought. To do otherwise would be ‘wasting time’. And yet although I dislike this kind of task, there is sometimes a place for sharpening your tools by repeating a process until you have an instinctive feel for it. So I am always looking for ways for pupils to practise procedures while something a bit more interesting is going on – even if it’s in the background.

As I was thinking about this, my Y9 class were substituting small positive integers into linear, quadratic and cubic expressions of their choice, producing sequences and then trying to work out how to go from the sequence of numbers back to the formula that generated it. This was going reasonably well, and I became interested in the idea that any finite sequence of numbers could be described by a sufficiently complicated formula. (So those questions that give a sequence and ask for the next number could theoretically be answered with any number.) It took all of my lunch-break to come up with a formula that gave the digits of the school telephone number for $n = 1, 2, 3, 4, 5, 6, 7, 8$:

$$u = \frac{107n^7}{5040} - \frac{241n^6}{360} + \frac{1547n^5}{180} - \frac{4157n^4}{72} + \frac{156179n^3}{720} - \frac{81017n^2}{180} + \frac{197387n}{420} - 180$$

“Whatever you’re doing must be very difficult because you’re using pen and paper, a calculator and a spreadsheet”!

The effort was apparent to my colleagues: one commented that “whatever you’re doing must be very difficult because you’re using pen and paper, a calculator and a spreadsheet”! It seemed fairly pointless, but I was proud of it, I knew that the class would appreciate the novelty, and I wanted some way of sharing it with them.

At the start of the next lesson I put it on the board as they came into the room. Several asked, “Is that for *us*?”, “Are we going to be doing that?” I nodded, and received either pleasant surprise or looks of horror! Once the class were seated, I said that I wanted us to investigate this equation, but that I felt that it was too much work for any one individual in the time we had, and I asked for suggestions of how we could tackle it as a class. There quickly arose two possibilities: each pupil/group could work on a particular term and evaluate it for different values of n, or each pupil/group could evaluate the whole expression for a particular value of n. We settled on the latter approach and I guided the discussion towards small positive integer values of n. It was felt that it would be better if pupils working on the same value of n were not sitting together (to aid cross-checking), so someone suggested that I allocate values of n around the room, which I did. And the work began.

The first problem was that some pupils had not brought calculators, so they became roving consultants, helping anyone who was stuck or supervising the collection of results on the board. Pupils wrote up their values as they got them, ticking the value already there if they agreed with it. One pupil appointed himself supervisor, and kept the area directly in front of the board clear of spectators! He also suggested the next value of n for people to try whenever there seemed to be contradictory answers.

Pupil-pupil and pupil-teacher conversations centred on applying BIDMAS (in this case, powers, then multiply, then divide, then add or subtract) and where to find buttons such as power and memory on the calculator. A frequent enquiry was a pupil holding up a calculator display such as 2.99967365 and saying, “Is that supposed to be 3?” I was reluctant to help: “If you think it ought to be 3, why isn’t it 3?” Adding up the rounded values of each term was a common problem. Some pupils tried working with exact fractions.

One pupil proudly declared that he could work them out instantly in his head, proving it for $n=0$ ($u = -180$), but declining to repeat his trick! So he added an extra column to the table to accommodate his answer. Others were less enthusiastic about the task, at least initially. One normally competent pupil was unable to begin because, I think, she was daunted by the complexity on the board. She kept saying that she “didn’t understand”, although she could repeat to me what the task was. She eventually agreed that she could work out the first term with her value of n and write it down, and once she had done this she had got over the barrier and was soon writing her value of u on the board.

Excitement grew as the table filled up. As time was running out I called a halt to the work and asked a pupil to read out her best guess at the numbers (there were still some conflicting values in one case). I asked the class to consider the significance of the values. After some time, the first two digits gave it away as a local telephone number. Someone had a mobile phone and wanted to try it, but before he could another pupil had confirmed from his homework diary the conjecture that it was the number of the school office. Some pupils tried for homework to construct a formula for their dates of birth.

Beforehand, I questioned the value of the task and nearly didn't do it, but persuaded myself that we'd had some hard lessons recently and it was harmless enough for a Friday afternoon. But with hindsight I think it was valuable for the class to work on something that initially seemed highly challenging but could be tackled successfully co-operatively. Pupils saw that a mathematical formula can produce a completely arbitrary, apparently random sequence of numbers. And some pupils found out how to use the power button on their calculators.

Y8 AREA LESSON

Colin Foster

I used to think that 'open' tasks were simply better than 'closed' tasks, and I was constantly seeking ways of transforming a closed task (such as 'find the area of this shape') into an open one (such as 'draw me some shapes with an area of 24 cm^2'). But I have come to the conclusion that the best tasks are often partially open and partially closed. Any task must have some constraints, after all, in order to define it.

My interactive whiteboard was suffering from a blown projector bulb, and my planning had become unusable, so I was complaining to my Y8 class at the beginning of period eight about having only a small squared whiteboard to work on.

I asked them to come to the board and draw any shape with an area of 24 cm^2. I offered a 6×4 rectangle as an example, but said that I was sure that they could come up with more interesting answers. (I said that we would treat the squares on the whiteboard as centimetre squares although they were obviously much bigger.) Each person had to give the name of his/her shape and convince the rest of us that its area really was 24 cm^2. The restriction was that each shape had to fit into the space left on the board, so pupils had to modify their ideas in response to what others had drawn. ("It's like in *Scrabble*, where you have a really good word but there isn't anywhere to put it!") We soon had several triangles, two parallelograms, a trapezium and a concave decagon. A kite gave us pause for thought, as without Pythagoras' Theorem we couldn't find its area using $\frac{1}{2}$ base $\times$ height, and we had to draw a square around the shape and subtract two congruent right-angled triangles. But the pupil's gut feeling ("I don't know; I just think it's right") was confirmed. Shorter people began to realise that they had to make the most of the space left at the bottom of the board, although we did allow one boy to stand on a chair. Eventually someone suggested that my original rectangle "wasn't really that good" and it was rubbed out to make way for a far more impressive trapezium.

Several pupils stayed behind at the end of the lesson instead of going home to try to fill in the gaps. Although there wasn't room for it, one pupil wanted to show his 4.9×4.9 square.

So what made this a successful task was its combination of open ('any shape you like') and closed ('its area must be 24 cm^2 and it must fit onto the board without overlapping anyone else's shape') features. Also, I think, the fact that the constraints were so arbitrary. My board happened to be 23 squares high (I hadn't checked that beforehand) and had some part squares visible at the edges. I chose 24 as a number with several factors, but it was clear to the pupils that the constraints weren't there to protect them from complexity but to offer an interesting puzzle.

Colin Foster is second in the mathematics department at King Henry VIII School, Coventry.

I have come to the conclusion that the best tasks are often partially open and partially closed.