Emily's discovery

Colin Foster

At the start of today's Y7 lesson, Emily excitedly told me what she had been working out during registration in the morning. I got confused trying to follow it in my head so she wrote it down for me as she explained. Paraphrasing slightly, this was her explanation:

> *Suppose you're trying to work out 3×6. (I'm not suggesting you actually do it this way, but it's interesting.) The second number has to be double the first – otherwise it doesn't work.*
>
> *You work out 3×7 = 21 and 2×7 = 14. The 7 comes from adding 1 to the 6; the 2 comes from taking away 1 from the 3. You always add 1 and take away 1 like that.*
>
> *Then you count in to find the middle number between these two answers (21 and 14). There are two middle numbers (17 and 18) and you always take the bigger one – and that's the answer to 3×6.*

She showed me that it worked with another example:

To find 4×8,

4×9 = 36, 3×9 = 27

27, 28, 29, 30, **31**, **32**, 33, 34, 35, 36

So 4×8 = 32

This intrigued me, but I couldn't immediately see whether it would always work or not. "I want to sit down and think about it but I have to teach the lesson," I said, and she laughed because not being the teacher she was free to continue with the problem: she had moved on now to products of numbers where the second was three times (rather than twice) the first but was not finding any pattern. The rest of the class were already mostly getting started with another piece of work, so I chose not to interrupt them to investigate this problem together – I also feared that it might not lead anywhere interesting. With hindsight that may have been a mistake. Perhaps it does no harm for pupils to experience the occasional blind alley – they appear in every area of thought, and fencing them off is keeping an important part of reality away from our learners.

Consequently, it wasn't until after the lesson that I had a chance to look at the problem myself, and fairly quickly I wrote down the following:

$$x2x = 2x^2$$

$$\frac{x(2x+1)+(x-1)(2x+1)+1}{2}$$

$$= \frac{(2x+1)(2x-1)+1}{2}$$

$$= \frac{4x^2 - 1 + 1}{2}$$

$$= 2x^2$$

This convinced me that Emily's process would always work, but of course these lines of algebra would not be illuminating to Emily! I was struck by how she could notice something that, to me, looking at it algebraically, seemed pretty complex and very hard to spot. I don't think I would ever have discovered it.[1] Yet I had the machinery to explain it more easily than she could. Perhaps this is a little like how a science teacher feels when a pupil without much knowledge of physics, say, notices some natural phenomenon and wants to know how it works? Yet a good science teacher can find a way of dealing with complex systems in accessible ways. Could I do something similar with Emily? I'm afraid I couldn't see how to make an accessible proof for her or to explain why the idea didn't seem to extend to the case of x multiplied by $3x$ (the 2 in the denominator not cancelling with a 3 in the numerator). Is this one of the things that contributes to the perception that maths is 'hard'? Is it the case that sometimes explaining *why* is simply out of reach for the present?

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1 I told this to Emily the next morning and said I was curious about how she had noticed her result. She replied that she was curious about that too – that it just came to her. Moments of insight seem often to be frustratingly hard to describe or explain.

Birthdays

Vivian Gussin Paley

Whenever children discover that a tall child is younger than a short child, they believe that somehow a mistake has been made. Bigger has to be older. I had known this for a long time, but I had not realised that you are bigger on the very day that you are older – on your birthday. On Rose's birthday she told us she was much taller and every child in the class agreed.

Teacher: Do you mean taller since yesterday?

Rose: Because I'm six.

Wally: Yesterday she was five. She *is* taller today.

Lisa: On that very day when it's your birthday you're bigger. If I was four and on one day I got to be five, I'd be much bigger.

Teacher: Then would you have to wait until your sixth birthday to grow some more?

Eddie: Otherwise how would you grow bigger?

Teacher: Rose, could I measure you? I'll put this mark on the board. Now, where do you think the line was yesterday, when you were still five?

(She puts a mark several inches lower. The others put theirs even lower.)

Teacher: Okay. Let's do this a different way. Kim, could we measure you? Pretend tomorrow is Kim's birthday. Today she is five and tomorrow she'll be six. Put a line at the place Kim will be when she wakes up on her birthday tomorrow.

(All the estimates are well above the original mark; some are six or seven inches higher.)

Teacher: Then must you get new clothes?

Eddie: Sure. Everything was too small on my birthday.

Lisa: When you start running it makes you have more energy. It makes you stronger and bigger every day. But on your birthday is when you grow inches.

Teacher: Fred, you're the only other child here who is six, so we'll ask you. We didn't see you on your birthday because it was during vacation. Did you *feel* taller on that day?

Fred: I really *was* taller. My bed was too short. I'm getting a new bed.

No matter how long the children discussed the question, the conclusion remained the same. It is as if there is a conspiracy to protect a communal belief from any contrary evidence. The teacher may wonder: Shall I bring out the ruler? I can prove beyond any doubt that a person does not grow instantly taller in one day. I will measure our next birthday celebrant the day before and on the day of his birthday, proving my point conclusively. The trouble is that children do not confer legitimacy on the ruler. We can insist that the children repeat our 'fact' – this brings them our approval – but we cannot force-feed concept before there is trust in the premise.

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