A formula for a square root

Colin Foster recounts a conversation with a student about square roots.

Pupil: "What's the formula for a square root?"

Me: "What do you mean?"

The pupil explained that she knew the formula for 'squared' – "you times it by itself". She was able to write this algebraically as $n^2 = n \times n$, in which she saw the right-hand side as an explanation of the function on the left-hand side. What she wanted was 'the answer' to $\sqrt{n} = ?$ At first she had thought, by symmetry, that it must be $\frac{n}{n}$ – if squaring is *multiplying* by itself, then the opposite must be *dividing* by itself – but that simply reduced to 1 – provided that $n \neq 0$ – so it didn't seem to work.

Me: "What do you think 'square root' means?"

Pupil: "You find a number which 'timeses' by the same number to make the number. So like the square root of 36, you think, and it's 6 because 6 times 6 is 36, but how do you get the 6 if you don't already know it? Do you always just have to guess?"

For me moments like this are wonderful. The pupil is asking questions in order to find sense in the mathematics, not because she is compelled to answer any question that I have set her or to achieve any particular externally-imposed end. But I am faced with a multitude of possible ways to respond. It seemed to me that she had uncovered the way in which inverse processes are often harder than the original. Dividing is harder than multiplying, factorising is harder than expanding, finding the centre of rotation is harder than rotating. Sixth-form students sometimes ask for something like a 'product rule' for integration and are disappointed that the best way to integrate is often to try something that might work and differentiate it and see. This does not always seem to them like 'proper maths': "I didn't take A-level maths just to guess answers!"

Of course, there are algorithms for finding square roots without a calculator, but I couldn't remember one off the top of my head. If I could have done, would it have been useful to share it with this pupil? I thought of drawing the graph of $y = x^2$ and thinking about inverse functions. I thought about formalising her 'just have to guess' into a more systematic trial and improvement process, perhaps on a spreadsheet. I thought about mentioning that $(-6) \times (-6)$ is also 36 or talking about imaginary numbers. Later, I thought about the Taylor series / Binomial expansion of $\sqrt{(1+n)}$ and about ideas of irreducibility and elementary functions. But none of these really felt like they were answering her question. Perhaps a more geometric response in terms of area would help? In the same lesson, another pupil was cubing and referring to it as 'triangling', because of the power of 3. When I asked him what he would call fourth powers, he said 'rectangling' and then was puzzled that it could just as well be 'squaring', as with a power of 2. In subsequent conversation, I realised that he had not connected the numerical process of 'squaring' with finding the areas of squares as geometrical objects.

In the end, I suggested that square rooting really is much harder than squaring, and admitted that although there are methods of square rooting by hand, I couldn't remember any, and am in just the same position as her when it comes to 'guess and try'. Rather than being disappointed by this, she enthusiastically set about 'trying to find a formula' for it, suggesting that it might make her famous! Should I have tried to prevent her? I left wondering whether this question perhaps sheds some light on why learners commonly make errors such as $\sqrt{36} = 18$. Some learners seem to persist in dividing by 2 in order to square root even though they are less likely to multiply by 2 to square – although some do that too. There is a definite process to go through to square, and at least dividing by 2 is a clearly defined and reasonably easily performed and remembered operation. It is hard to explain how to find a square root without beginning with the answer. Perhaps the main thing I concluded is that it is better to be genuine about mathematical difficulties, and to empathise with a pupil's insight, rather than to launch down a well-travelled path, however interesting the pupil might find this, if it is not really scratching where they itch.

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Laurinda Brown and Alf Coles also describe a learner seeing 'divide into itself' as the inverse of 'times by itself' – see page 46-49 – Brown, L. and Coles, A. (2008) *Hearing Silence: Steps to teaching mathematics*, Black Apollo Press