SMALL IS BEAUTIFUL AFTER ALL!

Colin Foster describes how larger graphs are not necessarily more accurate.

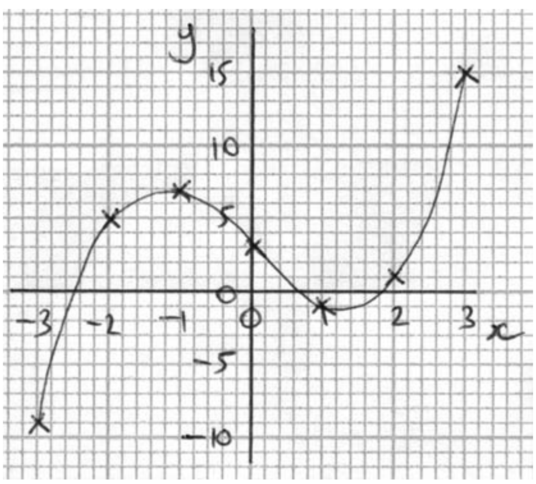
"Make your graph fill the page; the bigger you do it, the more accurate it will be." I've been saying this for years, but I now think that it may actually be rubbish. Leillah,[1] an environmentally-aware learner in my Year 10 class, always responds, "But it's a waste of paper!" and never draws a graph bigger than around 7 cm by 7 cm, when other learners might make theirs 20 cm by 20 cm, so as to take up most of an A4 piece of graph paper. I have often had one or two learners per class who have this preference for tiny graphs, and I have never really thought much about it. On this occasion, however, I couldn't ignore her approach.

The task involved finding the roots of the equation $x^3 - 5x + 3 = 0$ by drawing an accurate graph of $y = x^3 - 5x + 3$ and seeing where the curve crossed the x-axis.

The learners obtained the following table of values:

x	−3	−2	−1	0	1	2	3
y	−9	5	7	3	−1	1	15

Leillah's graph looked like this:[2]



and she obtained the solutions $x = -2.5$, 0.7 or 1.9.

The graph from one of her more obedient friends looked like the one on the page opposite. ... and they obtained the solutions $x = -2.55$, 0.50 or 1.70.

Leillah and her friend were considered to be equally 'careful' when it came to drawing graphs and, relative to the calculated values, sure enough Leillah did indeed have the more accurate solutions.[4] "*It's because I did it smaller, and smaller is more accurate – I told you!*" she said. Leillah felt that she could make the curve with more precision when she didn't have to move her hand so far. She could keep her hand in one place and sweep the pencil round smoothly; her friend had had to drag the pencil some distance, leading to a much greater 'wobble factor'. Joining up points a long way apart with a smooth curve isn't easy: when you watch learners do it, you see their eyes flicking back and forth from where the pencil is to the next point that they are aiming for; sometimes they pause along the way. However, her friend was surprised, and so was I, that the larger scale did not more than compensate for this. It was clear that Leillah not only saved paper, but avoided giving spuriously accurate answers, choosing to state her values to one decimal place rather than the two decimal places her friend felt confident enough to give, but which turned out not to be correct.

Being careful may be more important than being big, but I had always assumed that for a given amount of 'carefulness', bigger was more accurate. This is obviously too simplistic. Since this lesson I have looked for, and observed, the same thing happening several times. It would be interesting to analyse this statistically, although that would involve learners drawing the same graph many times on different scales, which they might be reluctant to do! It would also be interesting to compare the accuracy of alternative methods, such as drawing the graphs $y = x^3$, and $y = 5x - 3$, and looking for

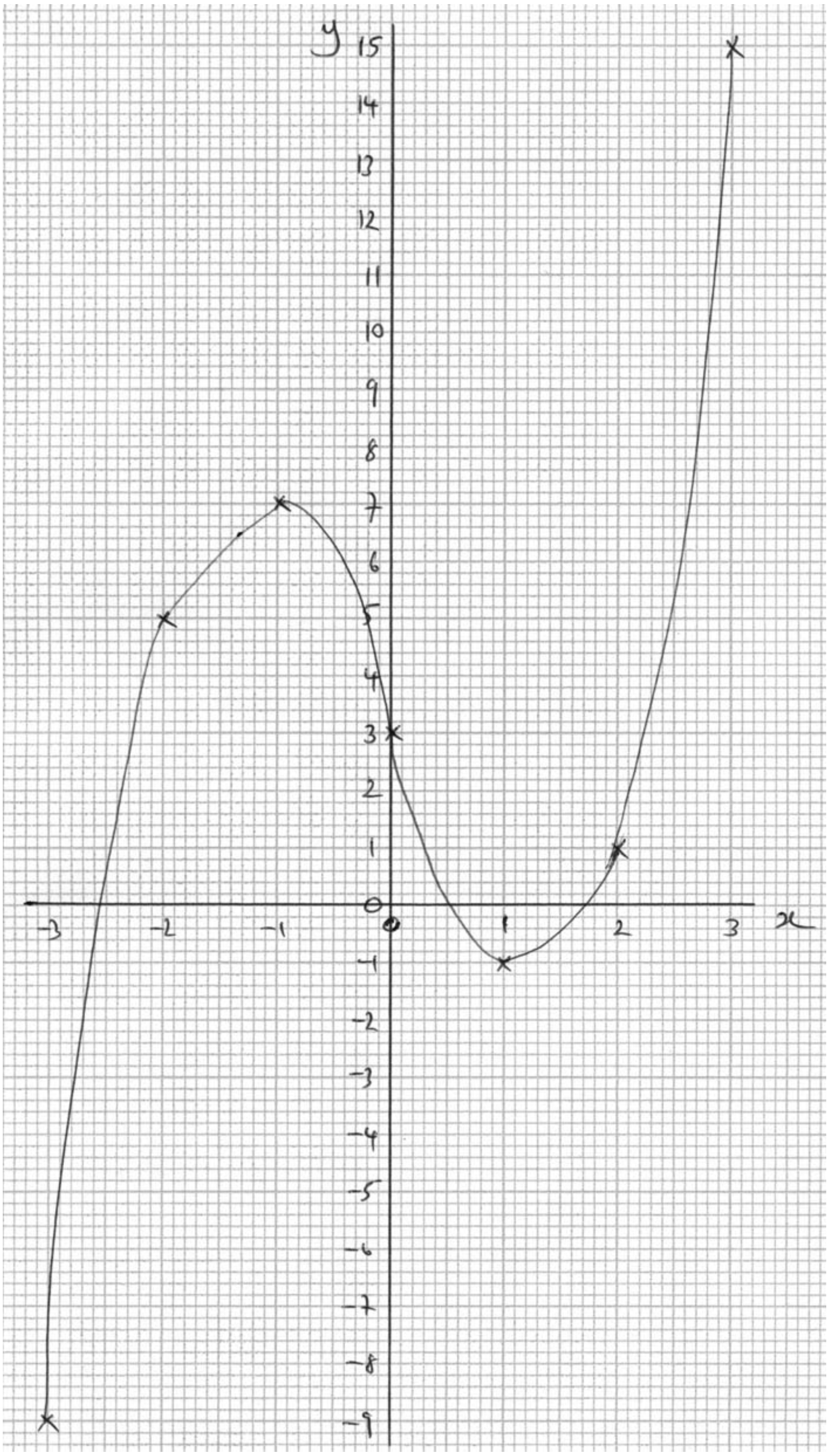
Notes

1 This is a pseudonym.

2 These two graphs are reproduced in the correct relative size.

3 In fact, it was too hard to separate 'carefulness' and 'scale' between different learners, so I redrew both of these graphs myself, drawing on the curves as carefully as I could. (It is my graphs and values that are reproduced here.) It would have been an interesting exercise to have the same learner draw the same curve on different scales, but understandably neither was willing to draw the graph again!

4 Calculated values for the roots (correct to 4 significant figures) are $x = -2.491$, 0.6566 or 1.834.



the *x*-values of the points of intersection. However, it is always nice when a learner's intuition proves to be more reliable than a bit of 'mathematical folklore'.

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