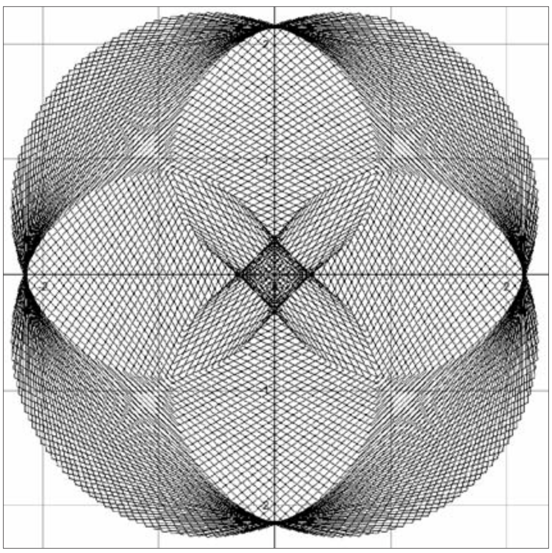
A PICTURE IS WORTH A THOUSAND EXERCISES

Colin Foster finds himself out of his comfort zone.

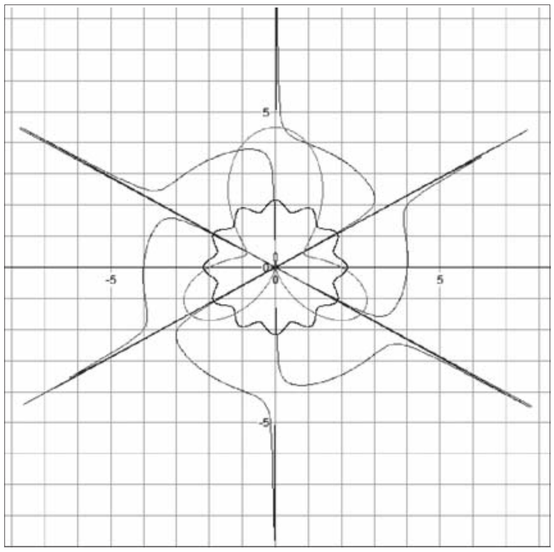
For me, one sign of a good task is that it will work just as well with Year 7 as with Year 13. Something I have done many times is to go into a computer room and ask learners to use graph-drawing software to '*draw a picture with graphs*'. The results are always startling and impressive.

Sometimes learners begin with equations that are familiar to them, which can provide interesting constraints, such as making pictures from straight-line graphs only. One way to start is to offer the equation of an ellipse, such as $(x-1)^2 + 2(x-3)^2 = 4$ and invite learners to draw it, and then to try changing the 1, 2, 3 and 4 and seeing what happens. Students may, or may not, decide to experiment with changing the powers of 2 as well. You can draw many things by using just ellipses of different sizes, shapes, and positions.

Eventually, though, the fun really starts when learners begin entering 'wild' equations and seeing what happens. Some of the outcomes will be suggestive, often of animals, objects, or letters of the alphabet, and then the challenge becomes how to modify these to make them more like what they already appear to be, for example "I need another eye, the same distance up but on the other side".



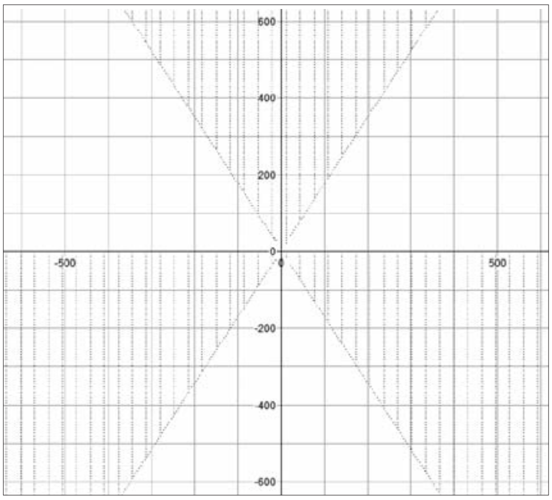
An insect's eye
$r = 5\cos\frac{\theta}{3}\sin\frac{\theta}{3}$ for 665 revs



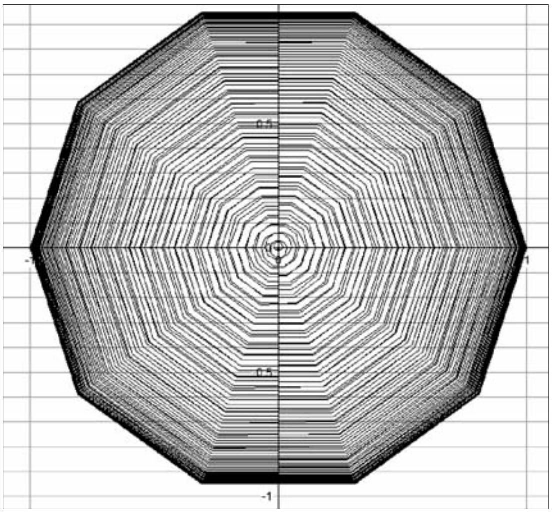
Symmetry
Can you tell which equation produces which curve?
$r = 2 + 10\sin\cos 12\theta$ for 1 rev
$r = 4 + 5\sin\tan 3\theta$ for 1 rev
$r = 2 - 1.5\sin 3\theta + \sin\theta$ for 1 rev

This involves learners in thinking about symmetry, transformations, estimation, scale, etc. They begin changing aspects of the equation and looking at the effects on the screen. This is one of the great affordances of ICT, that learners are in control, and the software obeys instantly, giving immediate feedback. Students don't need to ask the teacher what will happen or whether something is OK – they can just try it and look.

As a teacher, I am usually well outside my comfort zone of familiar curves, allowing me to be just as surprised by what is produced as anyone else. I see my role as using my imagination to suggest, if necessary, what a particular curve might resemble and helping learners to adjust their drawings to achieve what they are trying to make. Sometimes, one learner creates a picture and conceals the equation(s) and others try to replicate it, as near as they can.



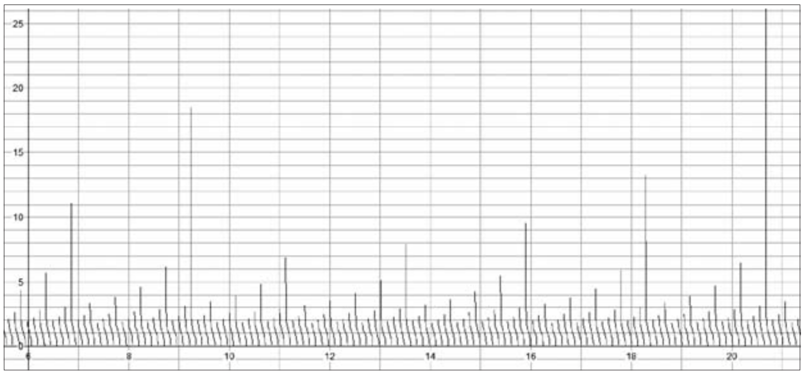
Triangles $(y-3)^3 + (y-2)^2 + x + 1 \geq 3x^2y$
Can you justify why this drawing has an order of rotational symmetry of 3?



Regular decagon
$r = \cos(1+\sqrt{\theta})$ *for 100 revs*

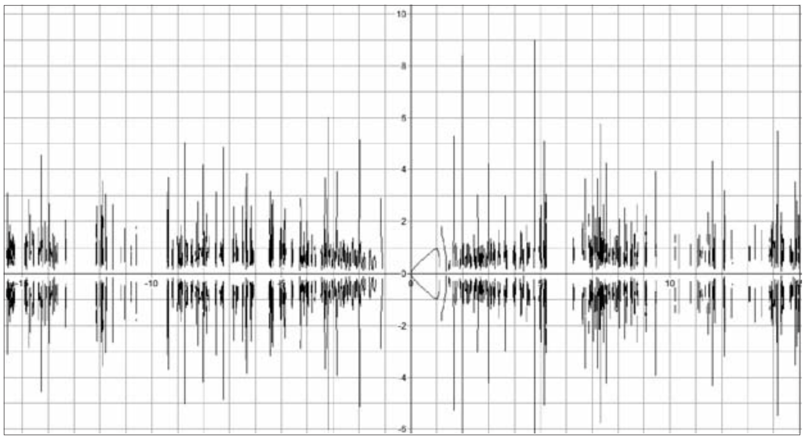
It has been fascinating to see the creativity, imagination and ingenuity of learners in this task. This spread contains an 'art gallery' of some of the things I have collected. You might like to consider some of these questions:

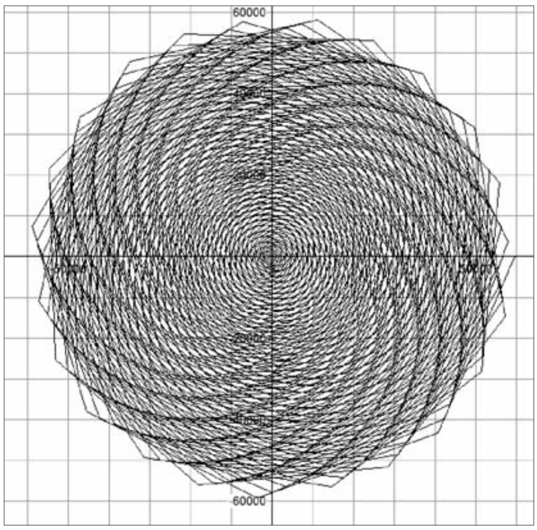
- Can you see why the equation(s) produce(s) the drawing shown?
  If there is more than one equation, can you see which bit produces which parts of the drawing?
- Do you think that there could be other interesting features of the curve that are not shown in the region displayed?
- Is there anything about the curve that you think is not true to the mathematical form of the equation but is an artefact of the way the software works? Why? Would the equation(s) display the same on a graphical calculator or on different graph-drawing software?



'Like a statistics graph' $y = \sqrt{1-\tan 25x}$
What determines where the bigger peaks come?

a calm still lake
$y^4 = \tan x^3 \cos x^5$





$r = \frac{4\theta}{\pi}$ *for 130 revs*

- What other questions can you ask about the curves?

Working in this way involves learners in a great deal of productive mathematical thinking and would seem to me to be worth any number of dreary exercises on 'transformations of graphs'; hence the title.

Colin Foster teaches mathematics at King Henry VIII School, Coventry.

Additional pictures can be found on the ATM website.