TASKS AND THEIR PLACE IN MATHEMATICS TEACHING AND LEARNING - PART 1

Jenni Back, Colin Foster, Jo Tomalin, John Mason, Malcolm Swan, Anne Watson set out to resolve the issue by 'doing tasks' in a variety of formats.

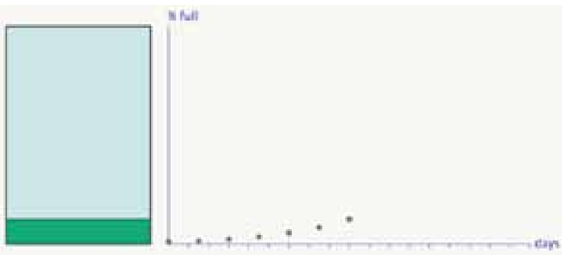
Every summer, John Mason, Malcolm Swan and Anne Watson run a residential four day workshop called the Institute of Mathematics Pedagogy. Up to 25 people, teachers, educators, and researchers meet and work on a theme about teaching and learning mathematics. In MT 226 the 2010 work was described, including a record of Jo Tomalin's own explorations. The next two articles describe the 2011 Institute which had the theme 'Tinkering with Tasks'. This theme was about how teachers develop and adapt tasks for use with their students.

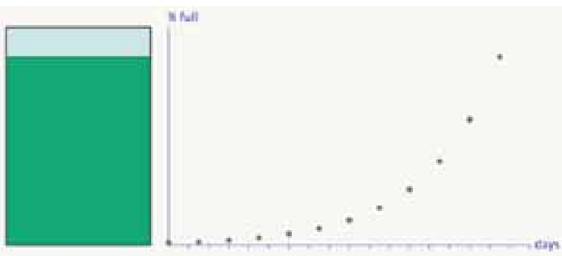
Examples of tasks

The Institute work always revolves around mathematical tasks, but in 2011 their design and characteristics were the focus of the discussion.

One of the tasks involved a story of duckweed covering Anne's pond. The question we considered was: 'how we might model the time it would take for the weed to cover over the pond'? given different rates of growth. We talked about how we might measure this, and how students of a range of ages might make sense of the idea and what mathematical concepts they might draw on to do this. Thinking about very young children understanding this scenario Jenni suggested that even quite small children have a notion of growth in an intuitive sense. In contrast to this intuitive level of understanding, John provided a dynamic model to represent the pond filling up with weed and a linked graph revealing the exponential growth. The applet permits several aspects to be varied.

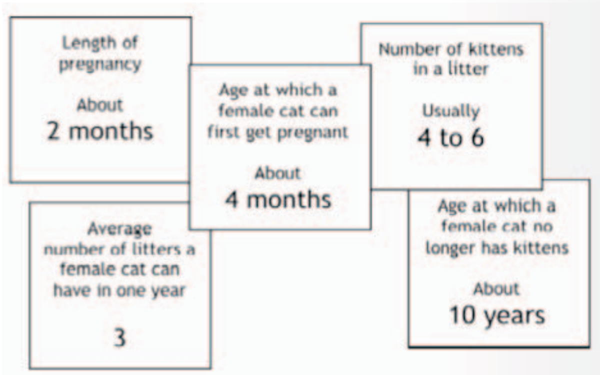
(the applet available at www.atm.org.uk/MT231)





Teachers need to help students to move progressively from their intuitive understandings of growth towards a formal presentation. In general it is important to build on children's intuitions about realistic puzzling problems so that mathematics relates to their own thinking and this might keep them engaged as they progress through school.

Another problem we looked at was the claim that one female cat could have 2000 descendants in the space of 18 months. The task was to comment on the claim given the following information:



Jenni: *The small group I was working with struggled with choosing appropriate notation and representation of our ideas, as well as the ideas themselves. What assumptions should we make? How could we show the offspring after different time intervals. All this work made me realise, yet again, how important and powerful the symbolic representation is, and how difficult it can be to access unless one has been involved in developing it, or is familiar with the approach to the representation.*

The relationship between task and mathematics

One of the big questions we considered was whether school mathematics can be separated from the tasks used to teach it. The correlation between task and its mathematical purpose has to be strong; the task shapes the learning; and we decided that it wasn't possible to adapt tasks, such as trying to make them more challenging or more accessible, without recognising how this might change the way students understand the intended concepts. Task design is therefore 'meta-mathematics' in the sense that it influences what is learnt, and how students understand the nature of mathematics.

One of the group suggested that there might be a helpful analogy between adapting and developing tasks and creating 'variations' in music. Variations in music might be construed to offer the performer an opportunity to show off their skill, or there might be a chance to challenge the performer or the listener, or they might explore the range of an instrument or develop the potential in a melody. Nevertheless there is a matching structure running through all variations, but if you only heard the very fancy versions, or the very slow versions, or those versions in a minor key, you might not grasp the underlying idea. How might we apply this to our development of mathematical tasks?

Some issues around this question were explored:

'the task looks familiar but there is still something new to be experienced because of subtle changes in the format'

'in designing a task how can I enable my students to see more than I see?'

'sometimes I can dislike a task (as a learner) and it is that discomfort that motivates me to make sense of it.'

Task adaptation

The participants at the Institute all brought a task that was in some way problematic and needed development. For most, this turned out to be a task with access problems; either its mathematical potential was inaccessible, or making it accessible seemed to dilute its potential. We found that rather than adapting tasks we were discussing the relevant pedagogy. Many of us were aware of school situations which required a very limited or stylised pedagogy, yet the answer to many of the problems we presented was to open up our pedagogy, such as planning only the first five minutes of the lesson and using our knowledge to orchestrate students' choices and contributions for the rest of the lesson, or sequence of lessons. The idea of *'theme and variations'*, or *'jazz improvisation'* made more sense than minute-by-minute planning if we were sure about several potential directions of learning from the task. Being sure about potential directions enables teachers to choose appropriate responses in the light of how they open up some possibilities, and close down others.

The purpose of tasks

Often when people talk about 'tasks' they are, in general, only referring to contextual situations, or potentially rich tasks, but we recognised that task purposes can vary hugely from introducing something new to practising a skill as well as to develop exploratory, problem-solving and creative approaches. What kinds of tasks might introduce a 'new' idea to learners? For example, the secondary curriculum introduces algebra as a tool for working at general and abstract levels rather than an ad hoc level, yet many extended tasks allow a trial-and-adjustment approach which depends on particular numbers, or the appearance of graphs. So we had some discussion about tasks which require an algebraic approach and, what is more, make it an attractive method such as making an algorithm for general use, or finding unknown values in a variety of cases.

Teachers who are not bound to a textbook choose tasks with a particular purpose in mind. We generated reasons for choosing tasks which range across mathematics.

Reasons about the nature of mathematics:

- to promote mathematical discussions
- to develop skills and stimulate mathematical thinking
- to develop different ways of doing mathematics to extend versatility
- to improve learners' fluency
- to help learners use mathematics in decisions they may have to make in their everyday lives and employment
- to identify variables and relations between them

Social or generic reasons:

- to develop awareness of working in a group and the value of a group dynamic
- to support learners to step outside their comfort zone
- to develop resilience/patience/creativity
- to develop skills of justifying, reasoning, communicating, explaining

Pragmatic reasons:

- to succeed in assessments

Good/bad

We understood more of the complexities involved in evaluating tasks and felt that some simplistic labels were unhelpful in deciding which tasks might be good to use, and which to avoid. In particular one of the group commented that we needed to be: *'Moving on from 'open tasks = good, and closed = bad' because it is all much more complicated than that...'* Someone else recognised that it takes a very discerning teacher to recognise the difference between interesting tasks and interesting packaging. We all felt that the only way to really get to grips with a task was to do it oneself either alone, or with a group of others – indeed that is how the Institute operates. Another key observation was that the way in which we use tasks with learners is as important as the task itself.

As Colin says:

So is there such a thing as a 'bad' mathematical task? We were asked to bring to the Institute a task which was "problematic in some way, one that you have used but are unhappy with, or which you are supposed to use but you don't see its potential or how to use it". I am familiar with things not 'working', whatever that means, in my classroom, but this does not come down solely to the task. The same task can lead to lots of eager activity with one class and fall completely flat with another, so I am reluctant to ascribe 'failure' to the task alone. How a task is used, by the teacher and by the learners, seems much more important to me.

So I question whether tasks can really be 'pathological' and need 'fixing'. Mathematics that is 'problematic' in some way is often more engaging and fun in a learning context and the same may be true of tasks. A DVD player that is capricious and temperamental is annoying in everyday life, because we are not interested in the thing itself, only as a means to an end, whereas if you are studying DVD players because they fascinate you, then one that is doing something strange is likely to be of much greater interest. So a possible conjecture would be that well-behaved mathematical tasks, with predictable outcomes and narrow topic coverage, are preferred by those who just want to get through the lesson and tick the box, whereas more awkward tasks are preferred by those who like playing around and going deeper? Having written this, I immediately think that it can't be that simple!

An example of a closed task:

" To get to the snack machine across a crowded room I walk 20 paces North, 5 paces East, 5 paces South, and then 15 paces East. If the room had been empty of people and furniture, how many paces could I have saved by walking directly to it?"

This is a problem to be solved; it is closed; it could be left after it has been answered; it requires choices and decisions about approach. We are not saying this is an amazingly challenging or deep bit of mathematics, but despite being closed it is neither trivial, procedural, or obvious, and it could be enriched to generate a class of similar problems, or to find the appearance of the movements in a mirror.....

As Colin says:

Open-endedness, for instance, is not entirely embedded in the task itself but depends on how learners choose to operate with it. I suppose the most open-ended task I can envisage would be something like: "Investigate something mathematical – if you want to." This might be so vague as to provoke nothing from many learners. Closing the task down to "Investigating graphs" might be enough for many learners to get going on something that interests them, right the way down to a more closed task such as "Sketch the line y = $2x$*", although even the most closed task can be interpreted in more open-ended ways by learners who are accustomed to doing that, who might see the '2' as negotiable, for instance, or explore different scales or look at different ranges for* x*. Just as it is not always necessary to ask a question in order to begin a conversation, for example a conversation at a bus stop might begin when someone makes an apparently closed statement such as "Looks like rain.", it is not always necessary to offer a specific task in order to stimulate worthwhile mathematical activity. Sometimes valuable mathematical thinking takes place as a result of a learner interpreting a task in a completely different way from how the teacher intended. Does that mean that the task has 'failed'? Ultimately, I want learners to take control of tasks, to tinker with them themselves, and to find what is interesting for them mathematically.*

Low threshold; high challenge

Procedural teaching is low threshold and low challenge if you can do the procedure, but high threshold and high challenge, or well-nigh impossible, if you cannot. Therefore much of our time was spent trying to work out how, apart from fluency, mathematical learning of new ideas can be made available for *all* through low threshold-high challenge tasks. Malcolm presented a sorting and matching task relating to sequences and their various representations. This can generate new knowledge by matching what is familiar and conjecturing and finding reasons for the rest.

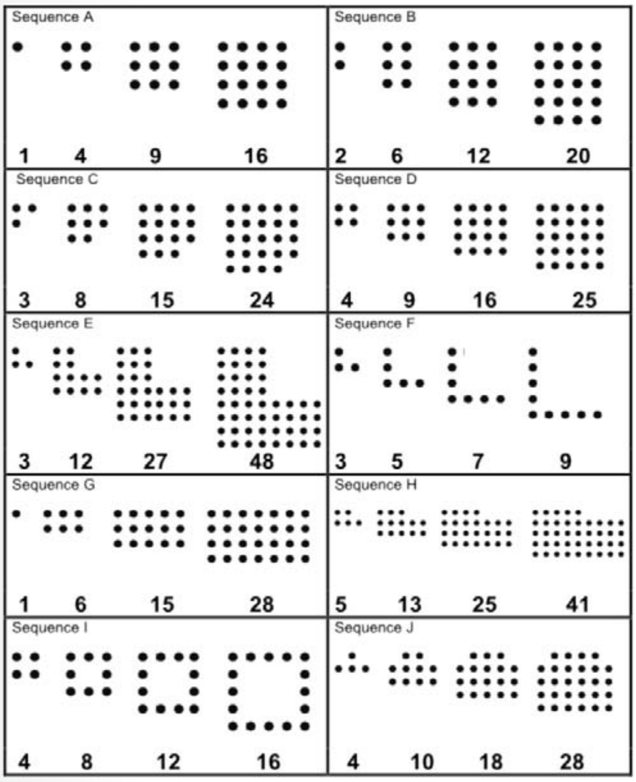
A "Sequences matching" task

We collected a number of further strategies from the group that might help develop tasks with a low threshold–high challenge. These included:

- read the question or task aloud and ask students to write down what they think is important before they begin on the mathematics
- ask the students to tweak tasks themselves
- use what students already know and what they can deduce from what they know.

Learners often get by in mathematics by trying to guess what their teacher wants them to think or do, so handing them the responsibility to choose or shape the task breaks that habit. Having a personal grasp of tasks can lead learners to experience the interest we had experienced in tinkering with tasks, but at a level that is relevant for them.

Card Set A: Dotty Sequences



Card Set B: Formulae for the nth term

n^2	$4n$
n^2+2n+1	n^2+2n
n^2+n	$2n+1$
$2n^2-n$	$3n^2$
n^2+3n	$(n+1)^2$
$n(n+1)$	$n(2n-1)$
$(n+1)^2-1$	$(n+1)^2-n^2$
$(2n)^2-n^2$	$(n+1)^2-(n-1)^2$
$n+n(n+2)$	$n^2+(n+1)^2$
$(n+1)(2n+1)-n$	$n+n(n-1)$

Card Set C: Number patterns

Sequence A

1st	$1+1\times0=1^2$
2nd	$2+2\times1=2^2$
3rd	$3+3\times2=3^2$
4th	$4+4\times3=4^2$
5th	....
6th	....
nth	....

Sequence B

1st	$2^2-1^2=3$
2nd	$3^2-2^2=5$
3rd	$4^2-3^2=7$
4th	$5^2-4^2=9$
5th	....
6th	....
nth	....

Sequence C

1st	$4-0=4$
2nd	$9-1=8$
3rd	$16-4=12$
4th	$25-9=16$
5th	....
6th	....
nth	....

Sequence D

1st	$4-1=1+2$
2nd	$9-1=4+4$
3rd	$16-1=9+6$
4th	$25-1=16+8$
5th	....
6th	....
nth	....

Sequence E

1st	$1+3=4$
2nd	$4+5=9$
3rd	$9+7=16$
4th	$16+9=25$
5th	....
6th	....
nth	....

Sequence F

1st	$3\times1=4-1$
2nd	$3\times4=16-4$
3rd	$3\times9=36-9$
4th	$3\times16=64-16$
5th	....
6th	....
nth	....

Card Set C: Number patterns

Sequence G

1st	$1\times1=2\times1-1$
2nd	$2\times3=2\times4-2$
3rd	$3\times5=2\times9-3$
4th	$4\times7=2\times16-4$
5th	....
6th	....
nth	....

Sequence H

1st	$1+1\times3=1+3$
2nd	$2+2\times4=4+6$
3rd	$3+3\times5=9+9$
4th	$4+4\times6=16+12$
5th	....
6th	....
nth	....

Sequence I

1st	$1+4=2\times3-1$
2nd	$4+9=3\times5-2$
3rd	$9+16=4\times7-3$
4th	$16+25=5\times9-4$
5th	....
6th	....
nth	....

Sequence J

1st	$1\times2=1+1$
2nd	$2\times3=4+2$
3rd	$3\times4=9+3$
4th	$4\times5=16+4$
5th	....
6th	....
nth	....

Pattern of your own:

1st	....
2nd	....
3rd	....
4th	....
5th	....
6th	....
nth	....

Pattern of your own:

1st	....
2nd	....
3rd	....
4th	....
5th	....
6th	....
nth	....

To prompt us to think about how we make sense of situations that are entirely unfamiliar to us, Anne borrowed a harness set from a friend who breeds Irish Dray horses. Tackling the problem of how the harness might be put on the horse so that it could pull a cart was challenging for us all. We had no sense of what different parts of the apparatus might be for other than dim memories from photographs or memories of riding as children. This is very similar to the situation that many of our students face when we present them with a challenging mathematics task. What are the goals of working on a task on which we have no experience to base our thinking, or only the vaguest outline of what mathematics 'ought' to look like? We explored our feelings of complete bewilderment and the way in which we moved towards tentative conjectures about the situation before trying things out. To our relief we did work it out.

This was partly through collaboration, and partly through understanding the final purpose so being able to test our solutions out against the expected use. The pictures show phases that came after the 'complete bewilderment'.

Malcolm says:

I was surprised to discover that the design of a harness for several horses pulling a carriage is equivalent to designing a mobile that you hang from the ceiling. Both involve considering turning moments. This 'making connections' with other problems is a powerful idea in mathematics, and one that maybe we should make more of in the classroom.

In the second part of this report we shall say more about the task adaptations we had learnt about through our experiences. If you cannot wait for that, do read Prestage and Perks (2001) and Swan (2006).

Jenni Back, Colin Foster, Jo Tomalin, John Mason, Malcolm Swan, Anne Watson

References

Prestage, S. and Perks, P. (2001). *Adapting and Extending Secondary Mathematics Activities: new tasks for old.* London: Fulton.

Swan, M. (2006). *Collaborative Learning in Mathematics: a challenge to our beliefs and practices.* London: National Institute of Adult Continuing Education.

Notes:

1. Announcements about the Institute appear on the NCETM website, or can be obtained at http://mcs.open.ac.uk/jhm3
2. This task was devised by Lesley Ravenscroft for Bowland Maths: See http://www.bowlandmaths.org.uk/
3. This was originally written by Malcolm for a US series: Mathematics Navigator, http://www.americaschoice.org/mathnavigator published by Pearson.
4. An applet for Duckweed can be found on-line at www.atm.org.uk/mt231

Reader Comment

I have appreciated the articles by Cosette Crisan in MT 209 and 230 about √16.

There seems to be something to be gained by the current ambivalent practice. Symbols are for our convenience, and the convenience may depend on circumstances.

You want the thought "square root" to be plus or minus

(a) when pupils are getting used to minus times minus makes plus;

(b) when solving $x^2 = 16$.

You want $\sqrt{a}$ and $a^{\frac{1}{2}}$ to be unique when

(a) you want an inverse to the function $x \to x^2$;

(b) you want the graph of $y = e^x$ to be a continuous line;

(c) you want to prove that $\sqrt[n]{a}$ exists for $a > 0$.

The formula for the roots of a quadratic equation would look odd with a special symbol for -√.

Bob Burn, Exeter University