TASKS AND THEIR PLACE IN MATHEMATICS TEACHING AND LEARNING - PART 1

Jenni Back, Colin Foster, Jo Tomalin, John Mason, Malcolm Swan, Anne Watson consider the format of tasks and what changes have the potential to optimise the outcomes for both teacher and learner. You may want to read part 1 in MT231 before continuing.

Every summer, John Mason, Malcolm Swan and Anne Watson run a residential four day workshop called the *Institute of Mathematics Pedagogy*. Up to 25 people, teachers, educators, and researchers meet and work on a theme about teaching and learning mathematics. Part 2 [Part 1 was published in MT231 November 2012] continues to follow the theme '*Tinkering with Tasks*'. The group worked together to consider how tasks might best be presented to optimise the learning opportunities and outcomes for all learners.

Tinkering with Tasks

This article follows on from the earlier exploration in Part 1 concerning the place of tasks in teaching mathematics, and draws on the work of the *Institute of Mathematics Pedagogy* held in Oxford in July 2011. We spent a great deal of time at the meeting looking at why and how we might adjust, develop, restrict, and open tasks and this article presents this to you.

As Colin said:

I came to the Institute thinking that tasks are one of the most important elements of what happens in the mathematics classroom. I still think that, but would qualify it by saying that how a task is used is much more important to me than the precise details of the task itself. I have seen mathematics teachers 'mine' great lessons out of what had appeared to me to be pretty uninspiring material, by using it to provoke discussion and inviting learners to critique and change it.

Why tinker?

There are a range of reasons we identified for '*tinkering*' with tasks and the phrase captures some of the sense of playfulness and fiddling that goes with the activity of adapting resources to use with students in the classroom.

The group suggested the following reasons for such '*tinkerings*' to make the task:

- more accessible
- deeper/richer
- include a surprise
- include more variety
- open up opportunities
- access different maths

Colin commented on the process of preparing to work with a specific task in the classroom:

'When looking at a potential task to use in the classroom it is nice to anticipate problems that learners might have yet not necessarily to try to eliminate them, but work on ways of supporting learners through the difficulties. For me, a task can only ever be a starting point, and soon after offering it to learners I find them doing things that I had not anticipated. I often find myself wondering whether I should 'bring them back to the task', like a politician who is failing to answer the question, but then why should I prioritise my agenda over what the learner is finding more interesting? I think I am growing to trust learners more to take what they need from a task. Human nutrition is immensely complicated, yet most of us believe that if we eat a sensible amount of a wide variety of foods, our bodies will naturally take the nutrients they need in the appropriate amounts. To try to plan for every vitamin, for example by giving each one separately, would be impossibly complex, and mirrors for me what happens when micro-learning objectives assume central importance in the classroom. If I offer learners a rich variety of mathematical experiences, perhaps I should trust them to take what they need from each in the quantities that they can handle at that time?'

Genres

Malcolm introduced us to five different genres of tasks that he has identified:

- *Evaluating mathematical statements*, where students are presented with an assertion that they have to exemplify or refute providing justifications and proofs. Always/sometimes/ never true activities are of this type.
- *Interpreting multiple representations,* where students interpret and translate between different forms of equivalent information. These often involve card matching, such as the sequences task presented in the previous article.
- *Classifying mathematical objects,* where students notice sameness and difference, identify properties and sort objects into categories then provide definitions.
- *Creating problems,* where students create problems for others to solve and then act as teachers to assist when the 'solvers' get stuck. This frequently focuses on doing-and-undoing, that is, operations and their inverses.

- *Analyzing reasoning and solutions*, where students are given some sample student work to analyze. These may be selected or designed to confront students with reasoning or methods they would not have thought of themselves.

This extends Pólya's simple distinction between '*problems to find'* and '*problems to prove'*, which applies to tasks seen from a pure mathematician's perspective.

Strategies
Having identified reasons why we might adapt or develop tasks, we considered strategies for doing so. Some of these strategies involved giving students a greater role in the process of working on them, such as swapping roles so that the student became the teacher, or letting students construct their own examples, questions, or tasks. Strategies for adapting and developing tasks might involve developing its presentation so that it became more, or less, precise in the information given, longer or shorter, or changed in form from a written question to a picture or audio visual presentation. Constraints might be added or removed, relevant or irrelevant information included or excluded, students bored with a slow strategy for solving something might be encouraged to look for a quicker or more efficient one, and we might use all these different approaches at different times. We discussed the need to create tasks where motivation was intrinsic: creating an itch, or baiting the hook. We talked about, and explored, the possibilities inherent on opening up text book tasks and helping students to analyse and develop them – these activities would help students to see the structure and point of questions more readily.

Context
We identified context as an important factor in presenting tasks to students. Contexts may help students to make sense of problems and they may motivate them by helping them to see its applications. This exploration led us on to questioning the relationship between mathematics and 'real life'. Do we leave real life once we start to generalise mathematically? For example, taking quadratics, we generally express a relationship in the form $y = ax^2 + bx + c$. What if we approach quadratics from sequences such as: 9, 4, 1, 0, 1, 4, 9 (subtracting 5, 3, 1, -1, -3, -5..); does this make quadratics more or less real, or more or less interesting, or more or less relevant to what learners already know? Different approaches to using context may work better on different occasions, and the group compared situations in which the context dictates the mathematics with those where the mathematics dictates the context. In the first, intuitions can be generated and used alongside a repertoire of known methods and concepts - the power of mathematics as a tool; in the second, the context is an illustration of the power of mathematics to describe. Some of us develop story contexts to dramatise the mathematics, and about how contexts could be fantastic, or real, as long as they were motivational. We were also very aware that some abstract mathematical ideas can be interesting and challenging in, and of, themselves. Another important issue is the one of choice. If we start with a problem from real life, then there will be many ways of solving it. If learners are given the choice of methods, then some will choose a numerical approach, while others may choose graphical or algebraic approaches. The lesson then becomes about comparing different approaches rather than illustrating a pre-determined area of content.

Tweaking
Moving beyond the contexts in which tasks are set we explored the effects of changing them and felt that small changes can lead to major differences in the outcomes and the responses of learners, known as the 'butterfly' effect, and we need to be aware of that potential as we make changes. We looked at general strategies that we used when adapting or developing tasks and came up with this list of approaches, see Prestage and Perks (2001) for our inspiration.

We can
- change the starting conditions
- include extraneous information
- decompose a task into 'layers' not steps
- reduce the problem to its bare essentials
- be less helpful in how we present the task, or more helpful possibly?
- remove information, for example measurements, but not so much as to 'kill' the problem
- take away vital information and require estimation
- remove someone else's mathematics, for example grids, tables, graphs, diagrams, or indeed add them
- remove sub-steps
- give students more time and longer tasks
- reduce the 'noise' to avoid cognitive overload by stripping the task back to its essentials
- give information in a way which focuses attention on different aspects of the problem situation
- offer learners the opportunity to generalise to the point of ridiculousness

Models and representations

Some of our work on tasks involved looking at models and representations and how they alter and affect learners' responses to tasks. Different representations give different access to ideas, or even to different conceptions of mathematical ideas, for example graphs, equations, and situations all give different perceptions of relations between variables. Representing situations is an important aspect of the development of mathematical thinking and reasoning and not easy to develop.

Our suggestions were:

- providing a simple model with its answer that might be used to model a more complex situation
- giving instructions or information in a way which focuses attention on different aspects of the physical situation
- asking students to think of different types of representation for an aspect of a problem
- drawing attention to the fact that models to think with are not necessarily models to apply, to predict, or control the world
- drawing attention to the idea that modelling is more than problem solving
- getting students to consider the degrees of uncertainty in the information they have

The different levels of mathematics in the models we use may vary from the very rough and ready, which we might use to answer questions such as 'which supermarket queue is the quickest?' through to those that are more overtly mathematical such as assuming the area of a octagon is close to that of a circle. This second model might lead to purely mathematical methods to find the volume of an octagonal prism. Our students need rough and ready alongside more precise methods to be fully numerate.

Task presentation & models of types of task

Another key factor in developing tasks to support learning in mathematics is the presentation of the task and its form. Multimedia can excite students. Teachers can develop variants of a task for learners who are struggling and offer them easier access points. They can ask extension questions that move students on further in their mathematical thinking. Teachers can turn all sorts of mathematical questions into tasks in particular genres that encourage mathematical thinking and active participation.

Malcolm offers a range of these approaches such as:

- card sorts
- asking sometime/always/never true questions
- giving answers and requiring questions
- offering examples of student work, either real or fictitious, as prompts and spurs to thinking
- breaking a larger problem into a sequence of tasks
- changing the order of subtasks and getting students to discuss the order
- changing the task from being for the individual, to being for a group to work on
- provoking discussion
- finding a real stimulus

There are further ideas in Watson and Mason (1998). One of the most interesting features of this Institute was that thinking so hard about task design and presentation led to lively discussions about our own pedagogic strategies. It has been found before that if teachers work on mathematical tasks together it is much easier to shift into talking about how you teach than if you set out first of all to compare teaching methods. All the remarks in these two articles came from participants at the Institute, so overall we have not presented a coherent, well-structured argument with any clear outcomes. Instead we have tried to give a flavour of the issues that arose, in the hope that they might generate some discussion elsewhere.

Information about the Annual Institute can be obtained at http://mcs.open.ac.uk/jhm3

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