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# TWENTY-ONE FOREVER!

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When it is somebody's birthday at work, they bring in cake for everyone and put up a notice alongside, making mathematical references to their age; for example, "I'm a perfect 28," or "In my prime at 29," or "Happy at 31." Sometimes it is more cryptic: "The only time I will be one more than a square and one less than a cube" or "A fifth power for the last time"! However, beyond a certain age, these remarks start to tail off, as colleagues become less candid about revealing their age, even in coded ways that only mathematicians are likely to appreciate. So we get more comments such as "21 again!". On seeing this, somebody was heard to mutter under his breath: "Mmm, mod 30." This got me thinking as to whether someone could give their true age for the first 21 years of their life and then legitimately claim to be 21 for every subsequent year (Figure 1).

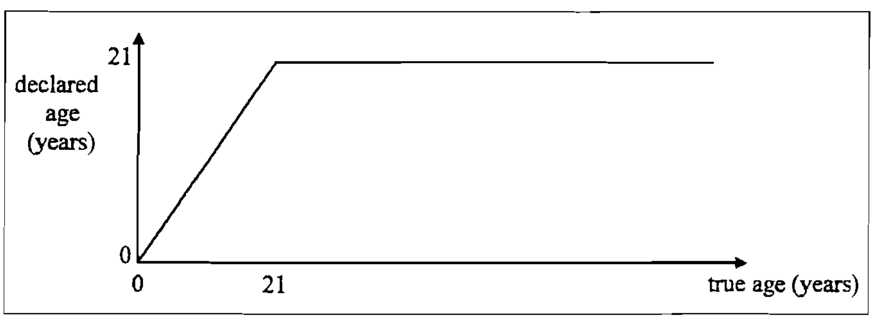


Figure 1. "21" forever!

The modulus trick works only from age 43 onwards, at which point, if you are $n$ years old, you can say that you are "21" mod $(n - 21)$. So, for example, when you are 48 you can say that you are "21" mod 27. This may be a nice way to deal with middle age and beyond, but what if you are uneasy during the intervening years from 22 to 42? One way of dealing with these is to use different number bases. Every odd age $n$ from 3 upwards can be regarded as "21" in base $\frac{1}{2}(n - 1)$ so, for example, 37 is "21" in base 18. To do this with *even* ages requires stretching things a bit, as you need rational bases. To be "21" at age 40, you need base 19½, which may be pushing the limits of plausibility somewhat. One solution to this would be to settle for calling yourself "22" in your even years, so you can use base $\frac{1}{2}n - 1$, which is an integer. That way your declared age would oscillate between 21 and 22 for the rest of your days (or at least up to age 43). So there are at least two ways to stay 21 forever—perhaps you can think of more?

All of this got me thinking about "charitable interpretations" of things that might otherwise be declared "wrong." When I started teaching mathematics, I was suspicious of the view that it is a subject that is always either right or wrong. I used to challenge myself to go a whole day without saying "no" to any mathematical statements that pupils made. It doesn't mean that if you say "no" then you are a bad teacher, but it can be interesting sometimes to replace "no" with "yes if." If someone simplifies $3x + 4$ to $7x$, you can say "Yes, if the 4 was $4x$" or "Yes, if $x = 1$." Mental work on times tables is perhaps one area where there does seem to be "only one right answer." I once saw a teacher trying to be positive when asking questions like "What is 7 times 7?" When a pupil gave an answer such as 54, he would say, "Ah yes, you were thinking of 9 times 6, weren't you?," which seemed a bit unlikely. But an eccentric teacher could reply to this answer by saying "Yes, in base 9." It would be an interesting challenge to see how often you could say yes to wrong multiplication answers:

$7 \times 9 = 53$ "Yes, in base 12."
$6 \times 8 = 44$ "Yes, in base 11."
. . . and so on.

For a quick-witted teacher, is this always possible?