THE PLAYGROUND!

Welcome to the Playground! Playground rules are posted on page 33, except for the most important one: *Have fun!*

THE SANDBOX

In this section, we highlight problems that anyone can play with, regardless of mathematical background. But just because these problems are easy to approach doesn't necessarily mean that they are easy to solve!

November's Sandbox contains two problems that are very simple to state. For **Problem 266**, **Rooting for You**, Colin Foster of King Henry VIII School (Coventry, UK) asks you to place the roots

$$\sqrt[2]{2},\ \sqrt[3]{3},\ \sqrt[4]{4},\ \sqrt[5]{5},\ \ldots$$

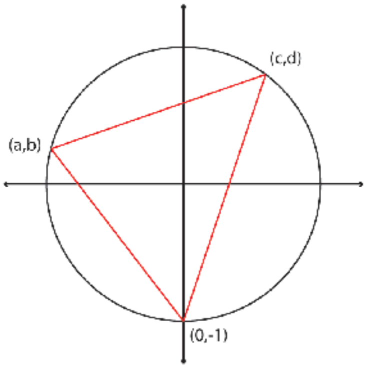
in order by size.

Problem 267, **Secant Squared**, comes from Herb Bailey (Rose-Hulman Institute of Technology) and W. C. Gosnell (Amherst, Mass.) and is the first of a series of triangle problems: do there exist isosceles triangles such that the length of each side is equal to the square of the secant of the opposite angle? One example is an equilateral triangle of side length 4, but are there others?

THE ZIP-LINE

This section offers problems with connections to articles that appear in this issue. Not all of the problems in this section require you to read the corresponding articles, but doing so can never hurt, of course.

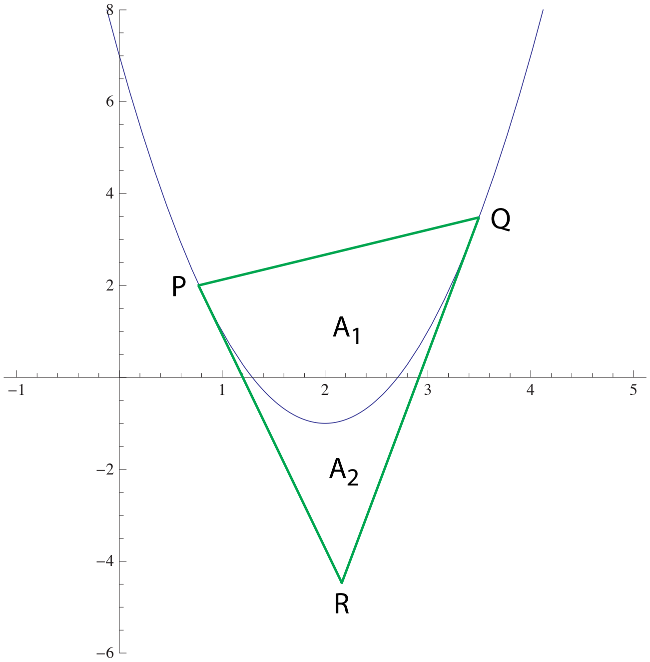
The article "Harvey Plotter and the Circle of Irrationality" by Nathan Carter and Dan Kalman on pages 10–13 presents a clever method for finding all points (x, y) on the unit circle $x^2 + y^2 = 1$ whose coordinates are rational numbers. November's Zip-Line problem, **Tri to Be Rational about This**, extends this idea by asking you to find triangles inscribed in the unit circle that are "rational" to the extreme. **Problem 268** is to describe all triangles on the unit circle with vertices (a, b), (c, d), and $(0, -1)$ such that the coordinates of the vertices and the lengths of the sides are all rational numbers.



THE JUNGLE GYM

Any type of problem may appear in the Jungle Gym—climb on!

November's Jungle Gym problem, **Archimedean Areas**, is from Michael Woltermann of Washington & Jefferson College. An Archimedes triangle of a parabola has two vertices, P and Q, on the parabola and the third vertex, R, at the intersection of the tangent lines from P and Q, as shown in the example below:



Archimedes knew that the area of the parabolic section of the triangle equals two-thirds of the triangle's area, so that $A_1 = 2A_2$. Let's say that parabolas have the "2-to-1 area" property.

Problem 269 asks about the converse of this statement: if a polynomial f has the 2-to-1 area property, must it be a parabola? To ensure that a triangle is created from any two points P and Q on the graph of the polynomial, let's assume that f satisfies $f''(x) > 0$ for all x.

This problem is really a Zip-Line problem with a long cable: Michael was inspired by the article Brian Shelburne wrote about "Archimedes and the Parabola" in the April 2011 issue, which considered the relationship between a parabola and a certain triangle inscribed in it.