References

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92.16 Avoiding Pythagoras

Readers will be familiar with the 'tilted square' method (Figure 1) of showing, without resorting to Pythagoras' Theorem, that the diagonal of a unit square is $\sqrt{2}$ units long. This can be useful with pupils who have not met the theorem, or for variety or, indeed, can be a means of introducing the theorem [1].

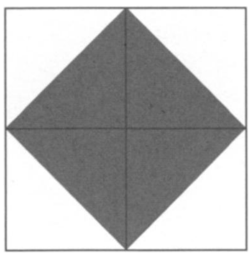

FIGURE 1

The area of the shaded square is four half-squares, that is 2 square units (taking the grid squares as unit squares). So the sides of the shaded square must be of length $\sqrt{2}$ units.

What may be less familiar (it certainly was to me) is that a similar procedure on an isometric grid can show, again without Pythagoras' Theorem, that the height of an equilateral triangle with unit side length is $\frac{1}{2}\sqrt{3}$ units (Figure 2).

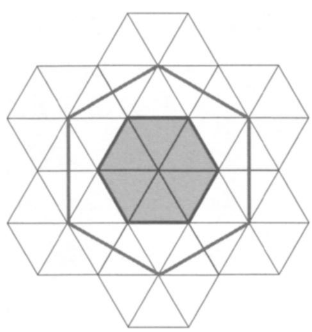

FIGURE 2

The central shaded hexagon and the large hexagon are both regular and therefore similar.

By counting small equilateral triangles, the area of the central hexagon equals 6 equilateral triangles and the area of the large hexagon equals 18 equilateral triangles. Therefore the areas are in the ratio 1 : 3, so their sides must be in the ratio $1 : \sqrt{3}$. So the lengths of the sides of the large hexagon are $\sqrt{3}$ units (taking the equilateral triangles to have unit side length), which means that the height of one of the equilateral triangles is $\frac{1}{2}\sqrt{3}$ units.

Reference

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92.17 Magic knight's tours for chess in three dimensions

The problem of a knight's tour on a square board (2-dimensional) is over 1000 years old and has a vast literature but its natural extension to three dimensions has received scant attention. Here, the knight is allowed to jump over the board and land up in a cell in a vertical plane. This increases the mobility of the knight considerably and the author has observed some interesting properties. Perusal of literature shows that Gibbins [1] had looked into the knight's tour in three dimensions. Gibbins asserts, 'The smallest lattice in which this (closed tour) can be done is a $3 \times 3 \times 4$ cuboid', that is 36 cells; he cites an example by E. Huber-Stocker, Geneva. The author disagrees with Gibbins because it is possible to construct closed, as well as open, tours in a much smaller lattice. In three dimensions, the author has observed that $3 \times 4 \times 2$ (24 cells) is the smallest lattice in which both closed and open tours are possible. This can be looked upon as two 3×4 boards, one above the other. Figure 1 is an example. Since there are thousands of such tours, they are of little interest. However, tours having magic properties are a different story. Figures 2 to 4 are such tours. The reader can visualise them in three dimensions by stacking the layers, one above the other, in alphabetical order. They have all the rows summing up to magic constant 50. This means that, since there are an odd number of terms in the columns, they cannot have magic properties. There are hundreds of such tours. If we consider a lattice with a square base then $4 \times 4 \times 2$ (32 cells) is the smallest lattice in which both closed and open tours are possible. The author has constructed sixteen interesting tours as shown in Figures 5 to 20. All these tours are magic in rows and columns with the magic constant of 66. Figures 6 and 15 have 8 pillars summing up to half the magic constant. They are shown in dark colour. Each of the 2×2 mini-squares also sums up to the magic constant. Here, open and closed tours are equal in numbers. The author proposes to call the tours from Figures 16 to 20 'a quintuple of tours' as they all have an identical layer and thus having an aesthetical appeal.