## 92.16 Avoiding Pythagoras

Readers will be familiar with the 'tilted square' method (Figure 1) of showing, without resorting to Pythagoras' Theorem, that the diagonal of a unit square is $\sqrt{2}$ units long. This can be useful with pupils who have not met the theorem, or for variety or, indeed, can be a means of introducing the theorem [1].

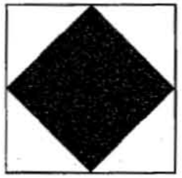

FIGURE 1

The area of the shaded square is four half-squares, that is 2 square units (taking the grid squares as unit squares). So the sides of the shaded square must be of length $\sqrt{2}$ units.

What may be less familiar (it certainly was to me) is that a similar procedure on an isometric grid can show, again without Pythagoras' Theorem, that the height of an equilateral triangle with unit side length is $\frac{1}{2}\sqrt{3}$ units (Figure 2).

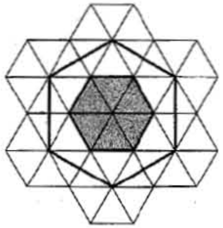

FIGURE 2

The central shaded hexagon and the large hexagon are both regular and therefore similar.

By counting small equilateral triangles, the area of the central hexagon equals 6 equilateral triangles and the area of the large hexagon equals 18 equilateral triangles. Therefore the areas are in the ratio $1 : 3$, so their sides must be in the ratio $1 : \sqrt{3}$. So the lengths of the sides of the large hexagon are $\sqrt{3}$ units (taking the equilateral triangles to have unit side length), which means that the height of one of the equilateral triangles is $\frac{1}{2}\sqrt{3}$ units.



*Reference*

1. Colin Foster, *Instant maths ideas: Shape and space*, Nelson Thornes (2003).

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**Avoiding Pythagoras**

Readers will be familiar with the 'tilted square' method (figure 1) of showing, without resorting to Pythagoras' Theorem, that the diagonal of the unit square is $\sqrt{2}$ units long. This can be useful with pupils who have not met the theorem, or for variety or, indeed, can be a means of introducing the theorem.[1]

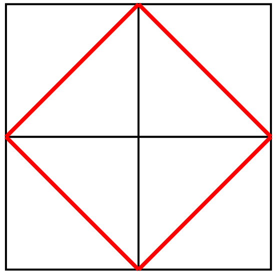

*Figure 1*

The area of the red square is four half-squares; therefore 2 units (taking the grid squares as unit squares).
So the sides of the red square must be $\sqrt{2}$ units long.

What may be less familiar (it certainly was to me) is that a similar procedure on an isometric grid can show, again without Pythagoras' Theorem, that the height of an equilateral triangle with unit side length is $\frac{\sqrt{3}}{2}$ units (figure 2).

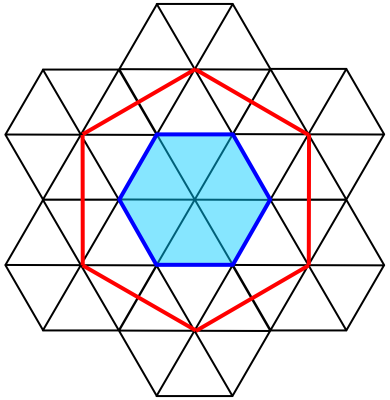

*Figure 2*

The red hexagon and the blue hexagon are both regular and therefore similar. By counting equilateral triangles, the area of the blue hexagon is 6 equilateral triangles and the area of the red hexagon is 18 equilateral triangles. Therefore the areas are in the ratio 1:3, so their sides must be in the ratio $1:\sqrt{3}$. So the lengths of the sides of the red triangle are $\sqrt{3}$ units long (taking the equilateral triangles as having unit side length), which means that the height of one of the equilateral triangles is $\frac{\sqrt{3}}{2}$ units.


*Reference*

1. Colin Foster, *Instant Maths Ideas: Shape and Space*, Nelson Thornes (2003).



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