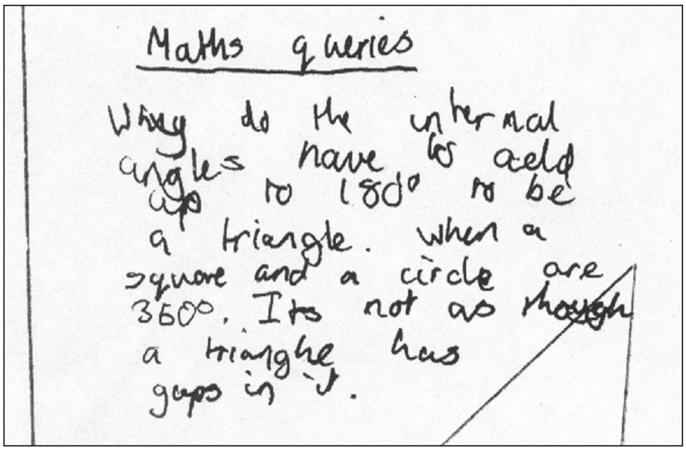
Questions Pupils Ask!

Asking Questions

by Colin Foster

As a rule, I don't go looking for trouble, so I am not in the habit of browsing through the back pages of pupils' books when I take them in! But in the course of searching for a piece of homework I stumbled across a question a Year 8 pupil had written (for himself, I presume) under the intriguing heading 'maths queries'.


Maths queries
Why do the internal angles have to add up to 180° to be a triangle. When a square and a circle are 360°. Its not as though a triangle has gaps in it.

'Why do the internal angles have to add up to 180° to be a triangle when a square and a circle are 360°? It's not as though a triangle has gaps in it.' (David)

To put this in some context, we had recently used the result that the interior angle sum of a triangle is 180° to find the interior angle sums of other polygons by cutting them up into triangles. We had also mentally 'walked round' various polygons and concluded that the exterior angle sum of any polygon has to be 360°.

I read and re-read David's (a pseudonym) query, trying to break into his thinking – a precarious business, but all I could do under the circumstances. When he assigns 360° to a circle is he thinking of a total *exterior* angle? Is he imagining walking right round the circle to the point where he started and realizing that he has turned through one revolution? Or is he thinking of a *radius* rotating through one turn? These thoughts led me to ask myself (I believe for the first time) what it might mean to think of the *interior angles* of a circle and, if it has any meaning, what their sum could be. Infinitely many vertices, each of 180°? The formula $180(n - 2)$ for the interior angle sum of an n-gon does indeed tend to infinity as n tends to infinity.

Does David see that all closed shapes ought to share this property (total exterior angle equal to 360°) and wonders how a triangle can join up if it doesn't? Yet a triangle *does*, like all polygons, have an exterior angle equal to 360°, though it is, perhaps, less often mentioned. Maybe the fact that a square (or any other quadrilateral) has an interior angle sum equal to the exterior angle sum of 360° is causing confusion? Or the fact that the interior angle sum of a triangle is exactly half its exterior angle sum? David's query has made me realize a number of potential confusions in this area that I was unaware of.

In the following week, some LOGO work in the computer room gave us plenty of opportunities to consider the differences between interior and exterior angles. Asking pupils to draw an equilateral triangle led many, predictably, to produce half a regular hexagon. Drawing a circle (well, a 360-gon and then a 3600-gon) satisfied me about my thoughts on the 'interior angles' of a circle.

My wish is that more of my pupils would question when a new idea seems to clash with the current structure they are carrying in their heads. As the teacher, I cannot reliably predict when and how this is going to happen. Being aware of common misconceptions has its uses, but can leave me trying to answer questions pupils are not asking. I believe that such clashes are crucial and can lead to fruitful opportunities for examining both the new idea and tweaking (or completely reworking) the existing mental structure to fit it. Perhaps we all need a 'maths queries' page?

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