Questions Pupils Ask!

Connected Expressions

by **Colin Foster**

I was working with my Year 7 class recently on solving equations. I had asked them to make up equations of the form:

$$\square x \pm \square = \square x \pm \square,$$

choosing numbers for the boxes so that the solution to each equation would be an integer. This had led to a lot of trial and error and plenty of practice at solving equations.

While they were working on this, Arun (Note 1) asked a question:

Arun: If you take away 3 from both sides of an equation, does that mean that altogether you've taken away 6?

I hesitated. I don't think I'd ever really thought of it like that before. Something made me feel uneasy about the direction this conversation might go in – it seemed like an 'unhelpful question'. I considered asking him 'not to think about it like that', but if that was how he *was* thinking about it then I felt obliged to go with it.

Me: I hadn't really thought of it like that. I suppose so.

Arun: I was just wondering: where does it *go*?

I had introduced a 'balancing' approach to solving equations through the use of the balance pan applet at the *National Library of Virtual Manipulatives* (Note 2), where things that are removed from either pan are dragged into a dustbin icon and just vanish. I considered whether this might have led Arun to wonder about this bin on the screen that was presumably getting fuller and fuller as we worked: who was going to empty it? (Note 3) (It occurred to me that Arun's regular responsibility as a 'recycling rep' to empty the recycling bins in some of the classrooms might be relevant here.) I had hardly noticed the bin myself, although I had used language such as 'get rid of the $2x$' and such like as we had worked together on solving equations; so on reflection, 'throwing away' had been part of the discourse all along. In fact we had also been *adding* things to both sides, and these had been obtained from an inexhaustible supply of 'x's and '1's on the screen, so Arun might also have asked where these were coming from, but didn't.

I felt that it might help to work on the distinction between *expressions* and *equations*. I have often encouraged pupils to indicate with curly arrows what operations they are doing to both sides of their *equations*, step by step, as they solve them. However, sometimes they then use these arrows inappropriately when simplifying *expressions*; e.g. in an extreme case a pupil might write something like:

$$\begin{array}{lll} & 3(x-2)+x & \\ & & \curvearrowright \text{ – brackets} \\ = & 3x-6+x & \\ & & \curvearrowright\ -x \\ = & 3x-6 & \\ & & \curvearrowright\ +6 \\ = & 3x & \\ & & \curvearrowright\ -3x \\ = & 0 & \end{array}$$

The equals signs suggest that they are simplifying an expression, which is keeping its value from line to line, but the curly arrows imply that they know that they are *altering* what they have in each stage. The first step blurs this distinction somewhat, since 'removing brackets' preserves the equality, and the pupil's notation of '– brackets' perhaps contributes to their problem. This hybrid method seems to be a confusion relating to the difference between an expression and an equation. In solving equations, the focus is on the fact *that* the two sides are equal, rather than what specifically they are equal *to*. This is particularly apparent when, for example, subtracting x from both sides: although we don't know what either side is equal to, and we don't know how much x is, we *do* know that if we take x away from both sides, the two sides will remain equal (provided that they were beforehand). This seems like quite a subtle point.

I reflected on subtraction problems, where 105 – 97, for instance, might be replaced with the easier calculation 108 – 100, by adding 3 to each number. It is well known that pupils sometimes worry about taking the 3 (or even 6) back off at the end. Similarly, the division $\frac{111}{6}$ might be simplified to $\frac{37}{2} = 18.5$, but pupils occasionally feel that they need to restore the factor of 3 that they divided the numerator and denominator by. The language used in some calculations can be unhelpful; for example, 'borrowing a ten' when doing a subtraction algorithm suggests the need to 'pay back' later on, yet what is really happening is that 'one ten' is being converted into 'ten units', and when I convert my pounds into Euros, I have no obligation to convert them back again in the future. For a Year 7 pupil, remembering to return things that they have taken can be a socially important aspect of adapting to a new school – even in the mathematics classroom we have rules such as 'If you borrow the glue, please put it back'– so they may have quite a strong feeling that it is wrong to take something without returning it. Pupils might also be familiar with situations such as 97 + 38, where they might calculate 100 + 38 and then subtract the 3 that they added on. In some situations, restoration is necessary and in some it isn't.

I decided to try to address the issue by having more than two expressions and to pair them up into equations in different ways, so that it became apparent that the expressions had not been 'destroyed' in the process of solving the equation. So next lesson I offered pupils the connected linear expressions diagram shown in Figure 1.

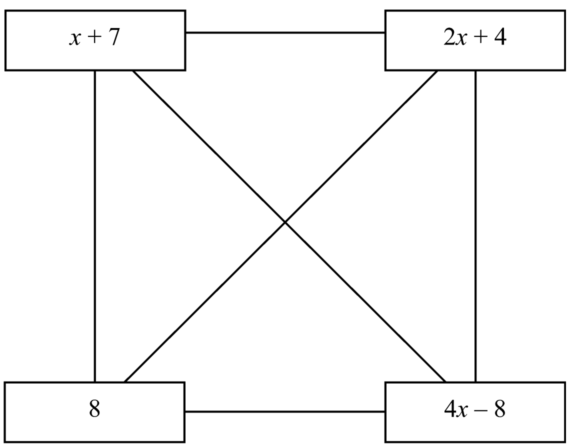

Fig. 1 Four connected linear expressions

The task was to write along each line segment connecting two boxes the value of x that would make the expressions in those boxes equal. Put another way, pupils were to solve the equations formed by equating each pair of expressions. The completed diagram would look as in Figure 2, with the solutions the integers 1, 2, 3, 4, 5 and 6. I invited pupils to make up their own ones with different expressions and perhaps with different numbers of boxes.

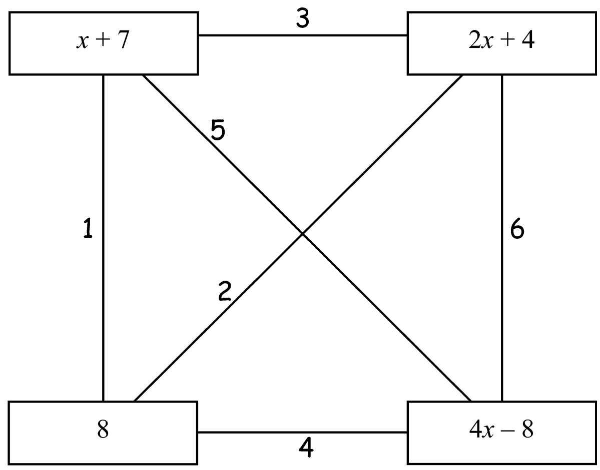

Fig. 2 The solutions: {1, 2, 3, 4, 5, 6}

Several possible questions emerged for investigation:

- Can you make others where all the solutions are *integers*?
- Can you make others where all the solutions are *distinct integers*?
- Can you make others where the solutions are 2, 4, 6, 8, 10 and 12, say?
- What is possible with more than four expressions, or with three expressions?

I also found it interesting to explore what happens when some of the expressions are quadratic.

Notes

1. A pseudonym.
2. **http://nlvm.usu.edu/en/nav/frames_asid_201_g_3_t_2.html**
3. In the related balance applet: **http://nlvm.usu.edu/en/nav/frames_asid_324_g_3_t_2.html,** negative quantities are represented by helium balloons, which can also be dragged into the dustbin. Having negative things in there might be thought to cancel out some of the accumulated 'positive' rubbish!

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