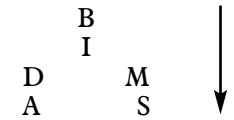
Higher Priorities

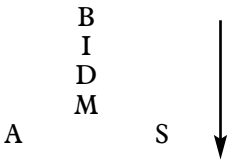
by Colin Foster

To help learners obtain the generally-accepted correct answers to calculations such as $2 + 3 \times 4 = 14$, several 'priority of operations' schemes are in common use, the most well-known in the UK being BODMAS or BIDMAS. But rather than working through brackets–indices–division–multiplication–addition–subtraction in that order, it is normally said that D and M have equal priority, as do A and S, and that the order within these is simply left-to-right as the calculation is written. This may be represented as below:



Equal priority is important; for example, $10 - 3 + 1 = 8$, not 6, even though A comes before S in the mnemonic. This aspect is frequently a source of confusion, often leading to incorrect answers. Perhaps it would be better always to place brackets in these cases, removing any ambiguity, but if BIDMAS is correctly understood and followed then such brackets should be unnecessary.

While no doubt other readers have noticed this long ago, it struck me for the first time this week that the 'equal priorities' notion for D and M is redundant. In other words, provided you carry out D before M you will always obtain the correct answer – although it is true that the calculation might appear a little more complicated. For example, $6 \times 4 \div 3 = 8$ whether you do the division first or work left-to-right (i.e. doing the multiplication first) although, admittedly, $6 \times \frac{4}{3}$ might be seen as trickier to evaluate than $24 \div 3$. However, it would seem that our scheme might be changed, without loss, to:



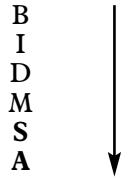
(Notice that this alteration would *not* work with the mnemonic in common use in the US, PEMDAS: parentheses–exponents–multiplication–division–addition–subtraction, where M comes *before* D. 'Equal priorities', or a swap round, *would* be necessary here. I assume that the only reason for the different ordering of the letters is to enable the 'Please Excuse My Dear Aunt Sally' mnemonic?)

This prompted me to ask whether the same might be true of A and S, *provided that S comes first*, and I think it is. You will never go wrong if you always do subtractions first;

e.g. $6 - 4 + 3 = 5$
and $6 + 4 - 3 = 7$;

both work out the same whether you go left-to-right or do the subtraction first.

Presumably the 'equal priorities' aspect has arisen because the letters BIDMAS make for a more pronounceable acronym than does BIDMSA, yet the latter would appear to be the simplest possible scheme, with a strict hierarchy and no messy 'equal priorities':



(note the order of the final two letters)

Perhaps all that is needed to make this workable is a catchy mnemonic, the following possibility being due to a colleague, not me: 'Boys In Dirty Macs Smell Awful!' (It is worth remarking, however, that a left-to-right process would still be needed, of course, where either subtraction or division is repeated, e.g. $12 \div 6 \div 2 = 1$, not 4, though in cases such as these, and especially with subtractions such as $10 - 2 - 3 - 4 = 1$, it would seem fairly obvious that left-to-right is the intended order, unless brackets are included, of course.

I think that order of operations is often *assumed* in school before it is taught explicitly, and, since it is a relatively arbitrary convention, this may be unwise. Mathematicians see at a glance that an expression such as $3 + 4a$ is equal to $4a + 3$, but the priority of multiplication is crucial to seeing why $3 + 4a$ does not equal $7a$, a common error. Pupils who make this sort of mistake are sometimes accused of lack of attention to detail ("It doesn't say $3\underline{a} + 4a$, does it? It says $\underline{3} + 4a$; you can't assume that a '3' is the same as a '$3\underline{a}$'!"). But often the error does not occur when the order is opposite ($4a + 3$), suggesting that the problem was not lack of attention but the pupil seeing $(3 + 4)a$. W. W. Sawyer wrote in 1964 about the difference between 'two egg and bacon' and 'egg and bacon, twice' (Sawyer, 1964, p. 101). When pupils in Key Stage 3 are beginning to face more complicated algebraic expressions, they are at the same time meeting quite different conventions in chemical formulae in science, so it seems to me that it is important to encourage the teaching of priority of operations earlier than is commonly the case. Perhaps BIDMSA might allow this to be done with less complication; in my view, BIDMSA needs a higher priority itself!

Reference

Sawyer, W. W. 1964 *Vision in Elementary Mathematics*, Pelican.

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