

# Questions Pupils Ask!

## Impossibility

by Colin Foster

There ought to be a mathematical topic called 'Impossibility'. Perhaps there is? I have never set out to 'do a lesson on' impossibility, but it became the centre of a recent lesson that was supposed to be about solving quadratic equations. A rather tedious bit of practice was brightened up when one pupil asked "What if it had  $x$  to the power of 3 in it instead of  $x$  to the power of 2?" I answered that there were methods for solving these algebraically too, but that by the time you get to equations containing  $x^3$  you can't make up a formula for the answer, and that someone had *proved* that it couldn't be done.

- Pupil: So if I worked out how to do it would I be really famous?  
Me: But you can't!  
Pupil: But if I did?  
Me: But you can't!  
Pupil: But what if I did?  
Me: You can't do it if it's impossible!  
Pupil: *Nothing's* impossible – it's just that no one's done it yet!

(I missed a trick here – I could have said that if it is impossible to find something that is impossible then we have found something that is impossible!)

I said that the mathematics of it was beyond me but that, as far as I understand, it is universally accepted among mathematicians that there is a perfect proof that you really *can't* do it, no matter how hard you try, and that no one will ever be able to. The entire class felt that this was very unreasonable and that no one can ever really say that something is definitely impossible because, who knows, they might manage it tomorrow? You've just got to keep trying and not give up! (It occurred to me afterwards that pupils had previously met the notion of impossibility in mathematics lessons only as the left-hand end of a 'probability scale' from zero to one. Whatever its merits, a probability scale reinforces the idea that impossibility is only one iota away from possibility, rather than its complete opposite.)

It is certainly a hard thing to accept. As I say, my mathematics is not up to the details, but I felt there was a point worth arguing for here, that some things really *are* impossible.

- Me: Lots of things are impossible, aren't they, and will never be done?

Pupil: Like what?

Me: Like make a triangle with exactly four sides.

Pupil: That's easy!

There was a lot of drawing, discussing and redefining of the term 'side' and some rather cute visual attempts, such as the one shown below, in which the pupil discretely held his finger over the corner:



removed to reveal a 4-sided triangle!

Pupil: Is this a triangle, Sir?

Me: I don't know; I can't see it all!

I was asked whether a triangle with two sides would do as well as one with four, and, once I agreed, a triangle cut out of paper was produced which, of course, has exactly two sides – a front side and a back side! There was also the old classic of 'It's got an inside and an outside!'

So what was going on here? Was this just an excuse to avoid the quadratic formula? Perhaps that was partly it, and no doubt it was a sign that we had been doing too much routine practice work recently, but there was so much sharp thinking and energy once the discussion began that it felt like the right thing to be doing at that moment. Pupils who might appear relatively unpromising mathematically when applying the quadratic formula and *still* not seeming to see that ' $b^2$ ' always has to be positive showed startling determination, originality and ingenuity. For myself, I hadn't realized before how important and potentially difficult is the idea that something really is totally impossible, no matter how cunning you try to be or who you are and what qualifications you might possess. "Even Mr X (a much-respected colleague) wouldn't be able to make a triangle with four sides!" suggested that they had begun to take on board the idea of mathematical impossibility! ☹

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