Problem 26

The total number of terms in the triangular array with n rows is $1 + 2 + 3 + \cdots + n = n(1 + n)/2$. From problem 10, we know that the rightmost (or last) term of this triangular array is $n^2 + n - 1$. But the row sums are $R(1) = 1 = 1^3$; $R(2) = 3 + 5 = 8 = 2^3$; $R(3) = 7 + 9 + 11 = 27 = 3^3$; ...; $R(n) = n^3$.

Hence, the sum of the first n cubes, $1^3 + 2^3 + 3^3 + \cdots + n^3$, is the sum of all the terms in this triangular array of odd integers $1 + 3 + 5 + \cdots + n^2 + n - 1$, which is an arithmetic series with common difference of 2. The sum of this sequence is

$$\frac{n(1+n)}{2}\cdot\frac{1+n^2+n-1}{2}=\left(\frac{n(n+1)}{2}\right)^2.$$

This result is in fact $(1 + 2 + 3 + \cdots + n)^2$.

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WHEN IS A PARABOLA NOT A PARABOLA?

The graph of $y = x^2$ is shown in **figure 1 (Foster)**. Or is it? Does anything strike you as not quite right about this image?

If we superimpose a genuine parabola (dashed), we see the difference (see **fig. 2 [Foster]**). The curve in **figure 1** matches at the origin and at the two endpoints of our parabolic segment but is too "pointy" in between. These "pointy parabolas" are everywhere—in mathematics textbooks, on the Internet, even on examination papers. Once you start looking for these pointy curves, you see them all over the place. The same problem occurs with cubics (see **fig. 3 [Foster]**) and with most other mathematical curves.

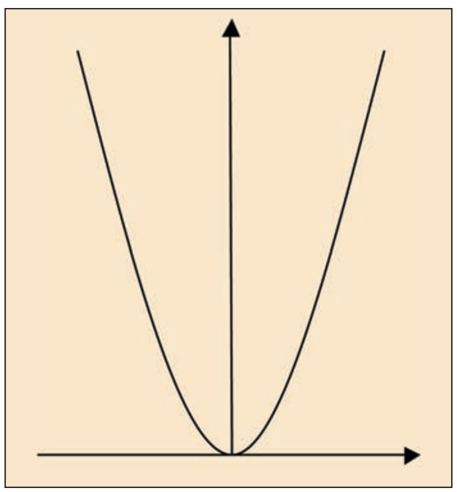

Fig. 1 (Foster)

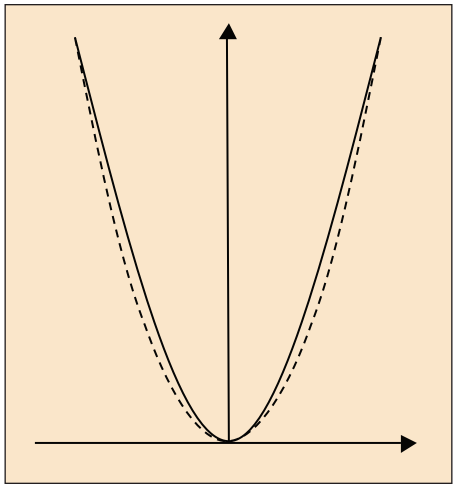

Fig. 2 (Foster)

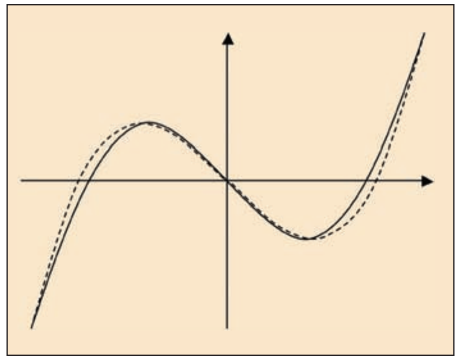

Fig. 3 (Foster)

These pointy curves come from computer drawing software, in which you click on a finite number of points (called "control points") and the computer creates a smooth curve linking them. In time gone by, the software used Bézier curves, and the curve did not necessarily pass through the points you clicked on (except for the first and the last). With three control points, these curves (quadratic Bézier curves) would produce a parabolic segment, so you could draw your $y = x^2$ without any problem. But it could be difficult to get exactly the curve that you wanted by dragging control points, which were often not on the curve. So more modern software tends to use "'curve through points" algorithms, producing such things as cubic splines, which lead to a smooth curve passing through all the control points, the position of the nth point determining the shape of the curve from the $(n - 2)$th point onward.

You may take the view that since these drawings are just mathematical sketches, it doesn't matter whether they are accurate; no one is supposed to be measuring them. However, by repeatedly using these inaccurate curves, we are in danger of systematically distorting some of the canonical images of elementary mathematics—standard curves that students should recognize as "friends."

This is a little plea for teachers to take a bit more trouble and draw curves using some of the excellent graph-drawing software now available. We shouldn't fob off "pointy" pseudo-parabolas on our students.

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