As Easy As 1, 2, 3, ... ?

They say that trouble comes in threes, but so do a lot of things. The 'rule of three' in public speaking suggests that audiences will latch on to a triad of parallel words or phrases: "Friends, Romans, Countrymen," "Blood, sweat and tears" – even "Education, Education, Education!" Three is the smallest number of items necessary for most people to detect 'a pattern'. If in front of a class you throw a die and get a five (say) and throw it again and get another five, there will be a slightly expectant hush. If you throw it a *third* time and get yet *another* five, there will be laughter. The second throw established the *possibility* of a pattern; the third throw *confirmed* it. Many jokes use this device to set up an expectation and then confound it in a humorous way (eg, Englishman, Scotsman, ... Irishman).

I see a relationship between this widespread phenomenon and what might be called 'linear thinking'. Two points are sufficient to establish uniquely a straight line – a third point merely acts as a check. Asking pupils what is the 'next number' in the sequence 1, 2, ... may result in the answer '3', but it is a bit of a shot in the dark. Given the start '1, 2, 3, ...', however, only the most awkward / creative will give an answer other than 4. But I think this approach requires serious challenge in the mathematics classroom. It is well known that school pupils often assume linearity when they shouldn't, whether it is lower school pupils assuming that 100 cm^2 equals 1 m^2 or A-level mechanics students using '*suvat'* equations when the acceleration is not uniform. It is dangerous to encourage simplistic expectations.

Furthermore, I would question to what extent pattern-spotting/ guessing is a mathematical activity. Professional mathematicians notice and study patterns, it is true, but these are patterns created by mathematical structure, which can be probed by the mathematician themselves. This is completely different from an invented pattern produced by another person according to a concealed and arbitrary 'rule'. Guessing the next number of such sequences is psychology, not mathematics: you have to try to work out what kind of pattern the other person is most likely to have created. This is no more a mathematical activity than is the game 'Numberwang',[1] which my pupils just lately have been constantly telling me about and asking whether we can play in lessons. The answer has been 'No', since, apparently (although I have never seen the comedy programme in question), it is a spoof quiz game where the contestants shout out random numbers until they guess the correct answer. Although it has numbers in it, it is not mathematical, and its pointlessness is presumably the joke.

In many schools, Year 8 pupils are required to sit a MIDYIS test,[2] in which one of the mathematics questions is

> What comes next?
> 1, 2, 4, 8, 16, ...

This is a particularly unfortunate example, since many pupils in our school will by that stage have met the well-known 'regions in a circle' investigation (see, for example, Foster, 2003, page 89), which leads, memorably, to the sequence 1, 2, 4, 8, 16, *31*, ... Some of the reasons for using this investigation with pupils are to suggest that patterns are not always obvious, that 'facts' are superior to anyone's 'theories', that first impressions are not always right, that surprise is an element of working on mathematics, and so on. The mathematical answer to the MIDYIS question is 'anything' or 'insufficient information', but since the test is multiple choice these responses cannot be given. The expected answer '32' depends on assuming a very limited imagination on the part of the examiners. An ability to notice or construct cleverer patterns will be penalised. There is something very wrong with an examination where to score highly you have to 'play dumb'. Such questions encourage an unhelpful approach to mathematics.

The MIDYIS test is designed to assess 'general ability' (whatever that means) rather than any kind of specific mathematical ability, but even mathematics examinations are not immune from this sort of problem. The following is an example of a question from a recent A-level examination paper, encouraging the same view of sequences:

> Sequences A, B and C are shown below. They each continue in the pattern established by the given terms.

A:	**1,**	**2,**	**4,**	**8,**	**16,**	**32,**	**...**
B:	**20,**	**-10,**	**5,**	**-2.5,**	**1.25,**	**-0.625,**	**...**
C:	**20,**	**5,**	**1,**	**20,**	**5,**	**1,**	**...**

> (i) Which of these sequences is periodic? [1]
> (ii) Which of these sequences is convergent? [1]
> (iii) Find, in terms of n, the nth term of sequence A. [1]

It seems to me that there is something wrong here. You cannot define an infinite sequence by listing the first few terms, waving your hand and saying '... and so on'. This is one of the first points made in this course when introducing inductive and deductive definitions of sequences. Why do the question writers choose to give the first *six* terms? Would the first term on its own be sufficient? Of course not. The first two terms? Still no, surely. The first three? After how many terms does the sequence suddenly become 'defined'? Is this not just a question of the limitations of your own imagination? There should be no place for 'Yeah – but you know what I mean' in mathematics exam questions! To say that the sequences 'continue in the pattern established by the given terms' betrays a disturbing naivety about possible 'patterns'. The answer to questions (i) and (ii) is that any of these sequences may end up being periodic or convergent; you cannot tell either of these things by looking just at the first few terms. There are, of course, infinitely many possible answers to part (iii), even if we were told (and we are not) that these lists begin with the *first* term.

Why am I making such a fuss about this when most people will answer this question without any trouble? Questions such as these cause no anxiety to students who have learned to expect to be shielded from complexity or with teachers who see exams as requiring conformity to what is generally expected and encountered: 'If something happens three times, it will obviously happen for ever'. This is inculcating a gullibility and lack of rigour when mathematics teaching should be doing the very opposite.

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Notes

1. 'Numberwang' is, apparently, a recurring sketch on *That Mitchell and Webb Look* (BBC TV). See http://www.bbc.co.uk/comedy/thatmitchellandwebbsite/numberwang/game.shtml.
2. MIDYIS is the Middle Years Information System, based at Durham University. See http://www.cemcentre.org/RenderPage.asp?LinkID=11410000.

Reference

Foster, C. (2003) *Instant Maths Ideas for Key Stage 3 Teachers: Number and Algebra*, Nelson Thornes