Questions Pupils Ask!

Odd Questions Pupils Ask

by **Colin Foster**

Sometimes the juxtaposition of two topics in a teaching schedule seems to provoke different questions and responses from pupils. I have often worked with pupils on odd and even numbers, but recently I did so with my Year 8 class shortly after some time studying negative numbers. They seemed to have enjoyed the negative number work particularly, with practical lessons walking up and down a number line and some peer teaching in groups, but I did not anticipate that many of them would come to the work on odd and even numbers with these thoughts uppermost. Sometimes it strikes me as an un-mathematical habit for pupils to assume that everything must be connected to whatever we have most recently done. No doubt this is encouraged if teachers habitually begin with a starter that appears to be unrelated to the main task but eventually turns out to be crucial. Perhaps we need more lessons in which ideas are not intended to form simple links in a chain? However, with the work on odd and even numbers, I was very pleased that when the pupils mentioned negative numbers they seemed to be making genuine comparisons between the two topics and thinking in highly mathematical ways.

Pupil: "Are there more even numbers than odd numbers, then?"

Me: "Why?"

Pupil: "Because zero is even."

We had had a lengthy debate during the negative numbers topic about whether zero was positive or negative. Some thought that zero must be positive by default, since it was not preceded by a negative sign. Others thought that it was positive because when you draw graphs in the first quadrant it appears on the axes alongside *all* the other positive numbers. Other arguments related to learning about zero early in primary school, long before negative numbers made an appearance. (The discovery of zero-as-a-number – rather than a placeholder – and also of negative numbers in the chronology of the history of mathematics is interesting here.) Some argued that because nothing was being removed, zero was definitely not negative, and so it must be positive. Others attempted to use the problems of division by zero to bolster their point of view. In the end there was a weak consensus that zero was the cut-off point between the positives and the negatives, and therefore was neither, although several were unhappy that this therefore meant that there were 'three kinds of numbers: positive, negative and zero', which seemed rather messy. One compelling image had been a number line from –10 to +10. The symmetry of having zero absolutely in the middle had convinced some that it must be 'neutral' and could not join either camp, unless it joined both! Had zero been included in the positives, there would have been 'more' positives than negatives, which somehow felt absurd – like there being more matter than antimatter in the universe (which, incidentally, is thought to be true in the visible universe)!

So now, since zero was firmly in the even numbers (a member of the 2-times table, since $0 \times 2 = 0$), it was no longer 'neutral', and so was swinging the balance towards the evens. This seemed very 'uneven'! Some notion of 'pairing up' was apparent in pupils' thinking, putting 1 with 2, 3 with 4, etc., and –1 with –2, –3 with –4, etc., seeming to leave zero out all on its own. But why not pair 0 with 1, 2 with 3, etc? Someone suggested that '$\infty + 1 = \infty$', so really adding 1 to infinity hardly made any difference and there were (about?) the same number of even and odd numbers. However, most pupils seemed to agree that although they couldn't say how many even or odd numbers there were, there was definitely exactly one more even number than there were odd numbers: zero.

When working with negative numbers we had developed the familiar multiplication table below, describing the sign of products of positive and negative numbers:

×	positive	negative
positive	positive	negative
negative	negative	positive

The neat symmetry of two positives and two negatives had been commented on. Indeed, for some, this had been sufficient to justify that 'two negatives make a positive', otherwise there would be 'too many' negative answers. It had been interesting to see how committed to ideas of symmetry the pupils had been. But now we had tables such as the following:

×	odd	even
odd	odd	even
even	even	even

This time there are three 'evens' and only one 'odd'! A pattern seemed to be developing: first, there seem to be more even numbers than odd numbers altogether, and now they are appearing in more than their fair share of cells in the table! The even numbers seemed to be winning everywhere!

The *addition* table for odds and evens is more equitable, with two odd and two even results:

+	odd	even
odd	even	odd
even	odd	even

When working on negative numbers, pupils had wanted to construct a table like this for the results of adding and subtracting directed numbers, but some had realized that the answers would depend on the *sizes*, as well as the signs, of the starting numbers, and so had reluctantly abandoned this. It is possible to do such a thing, but messy:

+	positive	negative
positive	positive	same as the sign of the number with the larger magnitude
negative	same as the sign of the number with the larger magnitude	negative

		b	
	a–b	positive	negative
a	positive	positive if $a > b$ negative if $a < b$ zero if $a = b$	positive
	negative	negative	positive if $\|b\|>\|a\|$ negative if $\|b\|<\|a\|$ zero if $a = b$

Pupil: How come odd numbers will (sometimes) go into even numbers but even numbers won't go into odd numbers?

The pupil had in mind divisions such as $\frac{even}{odd}=\frac{12}{3}=4$, compared with $\frac{odd}{even}=\frac{15}{4}=$ not an integer. Whereas $\frac{even}{odd}$ is *sometimes* an integer, $\frac{odd}{even}$ is *never* an integer.

There was an implicit assumption behind many of the pupils' questions and comments that odd/even should in some way parallel positive/negative, and much surprise that this analogy seemed imperfect. For me, this relates to experiences in science at school at about the same age, when I wanted north/south in magnetism to work the same way as positive/negative in electricity – and though there are definite parallels it is more complicated than a simple equivalence. The fact that even numbers are multiples of 2 whereas the odd numbers are not necessarily multiples of anything other than 1 and themselves (but may be multiples of other non-even numbers), makes the odds quite a different bag from the evens. Odds and evens are well-behaved under addition and subtraction, but they were never designed for multiplication!

Just as I was reeling a little from all these questions and observations, another pupil posed a question I had never considered:

Pupil: Are there any odd numbers that don't contain the letter 'E'?

Everybody began searching, but we couldn't find any, since 'even' special 'trick numbers' like 'googol' are even. However, someone wanted to know whether 'infinity' (with no E's) might be odd...

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