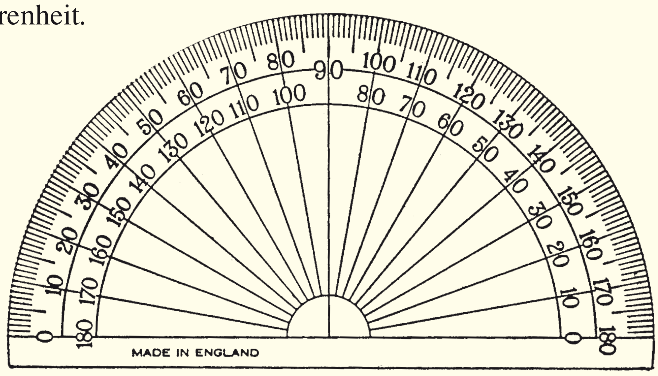
A DEGREE OF CONFUSION

As a teacher one can always rely on one's pupils to set one thinking.

Question: Why are there two scales of 'degrees' around the edge of a protractor?
Answer: One is degrees Celsius and one is degrees Fahrenheit.



This reply made me wonder whether there are any positions where this *is* the case (i.e. coincident numbers on the two scales which could represent the same temperature in degrees Celsius and degrees Fahrenheit).

When does this happen:

(a) on a 180° protractor;

(b) on a 360° protractor?

This is readily solved by simultaneous equations, using the fact that $F = \frac{9}{5}C + 32$, and of course the two readings must add up to 180, in case (a), or 360, in case (b).

The (C, F) values are (a) $(52\frac{6}{7},\ 127\frac{1}{7})$ and (b) $(117\frac{1}{7},\ 242\frac{6}{7})$.

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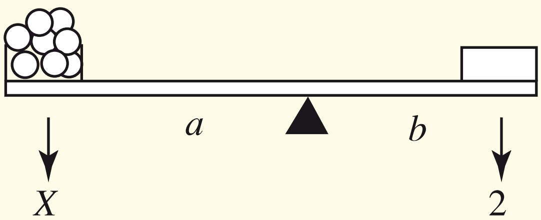
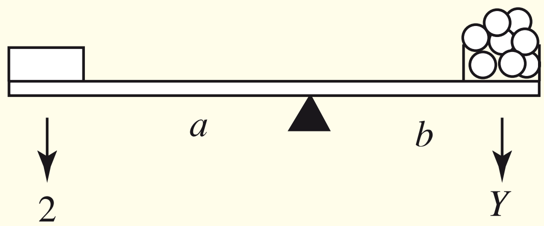
A MATTER OF BALANCE

School tuck shops are now required to sell 'healthy' food. Here's a problem from a long time ago (well, I found it on a Daily Puzzle Sheet from the MA Conference in 1988!), as you can tell from the units. Remember that there are 16 ounces (oz) to the pound, so 'a quarter' meant 4 oz.

"Our school inherited a curious old balance with the pivot off-centre. When a 'customer' asked for a quarter of peppermints the 'shopkeeper' used to first put a 2 oz weight in the right hand pan and balance it with sweets in the left hand pan, then put the 2 oz weight in the left hand pan and balance it with sweets in the right hand pan. The two amounts were then combined and sold as a 'genuine' quarter. Did the shop make or lose money?"





Suppose that the 'arms' of the balance are of lengths a and b, with the first weighing giving X ounces and the second Y ounces.

We thus have two equations: $2b = aX$ and $2a = bY$. So the 'customer' gets $X + Y = \frac{2b}{a} + \frac{2a}{b} = 2\left(\frac{a}{b} + \frac{b}{a}\right)$.

Now remembering that $(a-b)^2 = a^2 - 2ab + b^2 \geq 0$ we have $a^2 + b^2 \geq 2ab \Rightarrow \frac{a^2}{ab} + \frac{b^2}{ab} \geq 2$, since $ab > 0$.

Thus $\frac{a}{b} + \frac{b}{a} \geq 2$, with equality only if $a = b$, which we're told isn't so. So the 'customer' does better than the shop!

MLP