

## BUYING COFFEE

Here is a true story.

I was sitting in a coffee shop recently when someone arrived at the next table, bringing coffees for her friends, and saying, "No way! I ordered five but he only gave me four!"

One of her friends responded, "Are you sure? How much did you pay?"

The person who brought the coffees replied, "It was 12 pound something, and they were 2 pound something each".

What can you deduce from this? You might like to think about it before reading on.

(Thinking time ...)

Inequalities are a natural way to think about what we know here. The coffees may have all been the same price or different prices, but it does not matter.

We are told that

$$£2 < \text{cost of one coffee} < £3$$

and so we know that

$$£8 < \text{cost of 4 coffees} < £12$$

$$£10 < \text{cost of 5 coffees} < £15$$

because, even if the coffees were not all the same price, each coffee costs between **£2** and **£3**.

So, if the total cost was "12 pound something", then she must have bought more than four coffees, because four coffees priced at less than **£3** each cannot make more than **£12**. So she has over-paid!

Maybe this seems obvious? But it was not obvious to the people at the next table. It seemed to them that the problem required a calculator and a memory of the exact prices. Yet the power of inequalities is that precise information and detailed calculation is often not necessary. People often calculate things when an estimate would be sufficient. Working out  $4 \times 3 = 12$  is

easy, but realising that that is the relevant calculation to do is less so.

In fact, we can conclude a little more. Since we have deduced that she must have paid for 5 coffees, their average cost must have been between  $£12/5 = £2.40$  and  $£13/5 = £2.60$ .

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