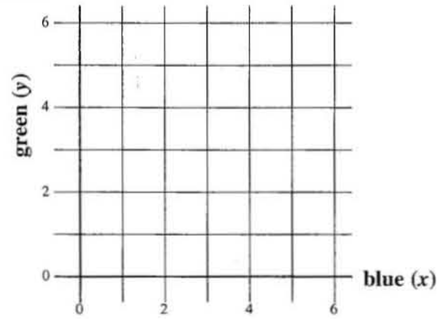
STRAIGHT DICE

I have two ordinary unbiased dice, one blue and one green. I throw them together and plot the point with coordinates (blue, green) on Cartesian (x, y) axes.



If I throw the dice three times, what is the probability that my points will make a straight line?

Answer

The simplest way to get a straight line is to get the same point twice together with a different point. Then we will effectively have only two distinct points, which is bound to make a straight line. The number of ways in which this can happen is ${}^{36}C_2 = 630$. (It is highly unlikely that we will get the same point *three* times – the probability of that happening is $\frac{1}{36^2} = \frac{1}{1296}$ – but if we do, we *won't* count that as making a straight line.)

If we get three distinct points then the only way they can make a straight line is if the gradient is $\pm\infty$ (vertical), 0, $\pm\frac{1}{2}$, ±1 or ±2. The number of ways in which each of these can arise is given in the table below.

gradient of line	number of possible triples leading to that line
$\pm\infty$	$6 \times {}^6C_3 = 120$
0	$6 \times {}^6C_3 = 120$
$\frac{1}{2}$	8
$-\frac{1}{2}$	8
1	$2({}^3C_3 + {}^4C_3 + {}^5C_3) + {}^6C_3 = 50$
−1	$2({}^3C_3 + {}^4C_3 + {}^5C_3) + {}^6C_3 = 50$
2	8

This gives us a total of **372** possible lines.

There are $36^3 = 46\,656$ possible outcomes altogether, so the probability of getting a straight line is

$$\frac{630 + 372}{46\,656} = \frac{167}{7776},$$ which is 2.1%, correct to 1 decimal place.

Is a probability of about 1 in 50 close to what you would have expected?

Colin Foster
King Henry VIII School, Coventry

Straight Dice
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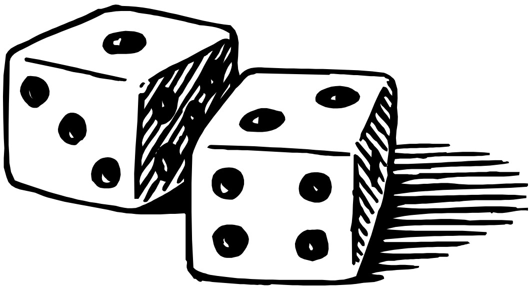

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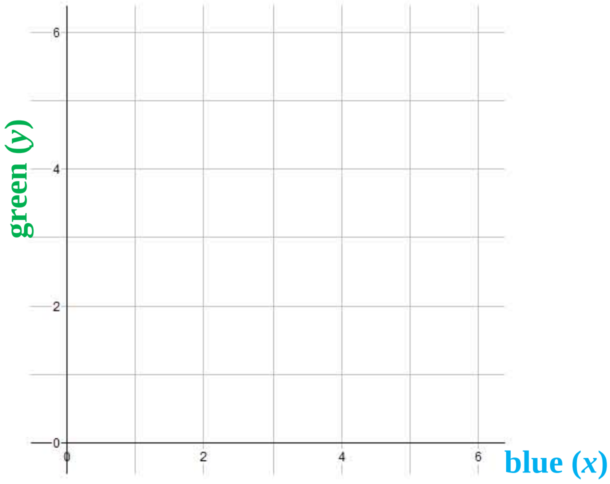

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2	8
–2	8
total	**372**

There are $36^3 = 46\ 656$ possible outcomes altogether, so the probability of getting a straight line is $\frac{630+372}{46\ 656} = \frac{167}{7776}$, which is 2.1%, correct to 1 decimal place.

Is a probability of about 1 in 50 close to what you would have expected?

Colin Foster teaches mathematics at King Henry VIII School, Coventry.