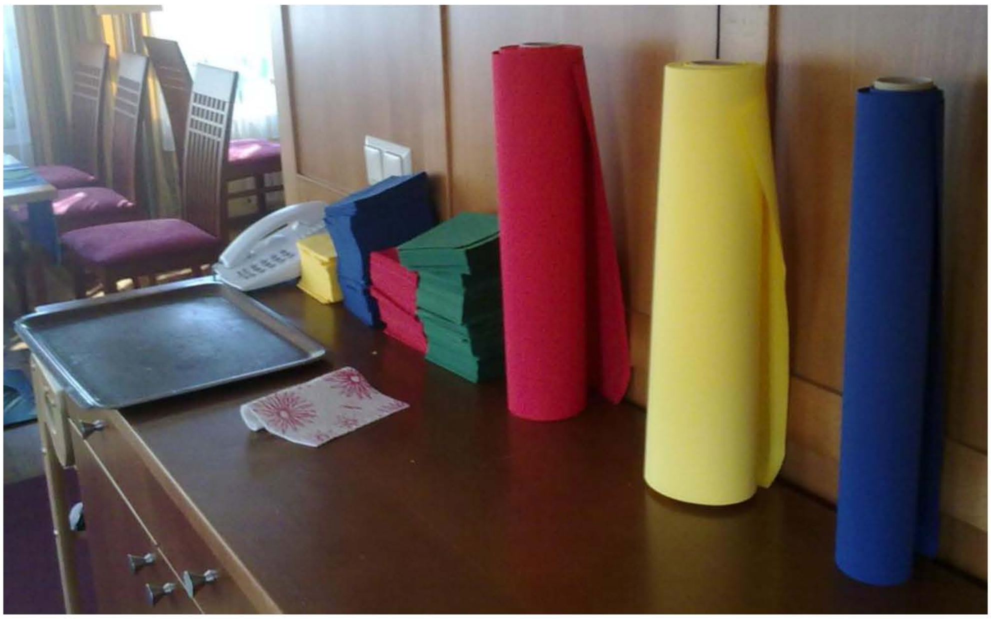
TABLE SETTINGS

On a recent holiday, I noticed that the breakfast tables at my hotel were being laid out with coloured sheets of paper across each pair of opposite positions (see photo 1).

Looking at where they kept the paper, I found that there were four different colours available: red, green, blue and yellow (photo 2). It seemed that no two adjacent pairs of guests were allowed to receive the same colour setting, but other than that repeats were permitted.



Could they, I wondered, set each table in the dining room with a different set of colours?

There are ${}_4P_3$ arrangements of three different colours from four, but we would need to count, say, red-yellow-blue as the same as blue-yellow-red, since this is simply viewing the same table from the other side. This gives $\frac{{}_4P_3}{2} = 12$ possibilities. However, arrangements with the same colour at both ends, such as red-blue-red, are also allowed, and there will be ${}_4P_2$ of these, which is also 12. So altogether there are 24 possibilities and, since there were fewer than 24 tables in the dining room, each table could be different – and was, as far as I could see.

Conveniently, in the other section of the dining room the tables were of length 4 rather than 3, and the same four colours of paper were available. How many ways are there now? This time, each table can have four, three or two different colours of paper. For four different colours, this leads to $\frac{4!}{2} = 12$ possibilities. With three different colours, we can arrange them as ABCA or ABAC, which can happen in $4({}_4C_2) = 12$ and $8({}_4C_2) = 24$ ways respectively. With two different colours of paper, we have to have ABAB, which gives ${}_4C_2 = 6$ possibilities. This gives a total of 54 ways.

Can you generalise to n colours of paper and tables of length r $(r \leqslant n)$?

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