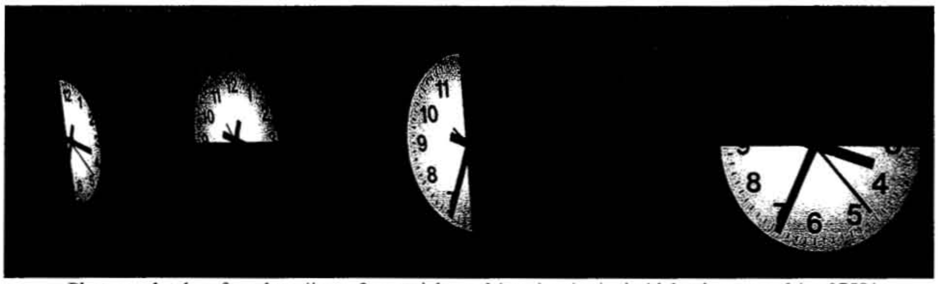
Symmetry Plus
Autumn 2007, 34, p4

THE WARWICK DIALS



Photograph taken from http://www2.warwick.ac.uk/services/art/artist/richardwentworth/wu0758/
We are very grateful to the University of Warwick Art Collection for permission to use this illustration.

These four eye-catching clocks, designed by Richard Wentworth, have operated at the Warwick Arts Centre since 2000, and prompt several mathematical questions:

- For the clock on the left, for how long in total during a 12-hour period are *all* the hands obscured by the black portion?
- Is the answer the same for each of the clocks?
- How long is the *longest continuous* period during which all the hands are hidden? When does this happen?

I am choosing to ignore the small portion of each hand that protrudes at the 'wrong end'.

Solution

Considering the first clock (the one on the left in the picture), it is clear that the hour hand is hidden for half of the time (six hours, from 6:00 until 12:00). For half of *this* time, the minute hand is also hidden, and for half of *this* time the second hand is also hidden.

So the total duration in which all three hands are hidden is $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 12$ hours $= 1\frac{1}{2}$ hours.

If you take an analogue clock that has only hour and minute hands (no seconds hand) and rotate the clockface with all the numbers 30° clockwise, the clock still functions correctly as a clock – it simply moves an hour back in time. When changing to and from British Summer Time in the UK, it would be just as feasible in principle, with a clock like this, to move the numbers round as it is to wind the hands forwards or backwards an hour. There is nothing special in the workings of a clock about the 12:00 position until you consider the second hand, which needs to coincide with the other two at 12:00. There is no other place where this happens (although the hour and minute hands meet 11 times – why is it eleven, not twelve? – in every 12-hour period). So if it weren't for the second hand, the answers for the other 3 clocks would be the same as for the first clock ($\frac{1}{2}\times\frac{1}{2}\times 12$ hours $= 3$ hours). However, when including the second hand, the answers for the other three clocks may be slightly different. The second and third clocks actually also come to $1\frac{1}{2}$ hours, but the fourth one is different. Is it more or less? By how much? Why?

The longest *continuous* period during which all the hands are hidden must be at most 30 seconds, no matter what the orientation of the black portion, since the second hand turns through 180° every 30 seconds. And this is the answer for each clock. For the first clock, it happens at 6:30:30-6:31:00, 6:31:30-6:32:00 and so on, 30 times every hour, so 180 times in every 12-hour period. The same will be true (although the precise times are more awkward to calculate) for the other three clocks.

Colin Foster
King Henry VIII School, Coventry.

A THOUGHT FROM ALBERT EINSTEIN

There is another reason for the high repute of mathematics: it is mathematics that offers the exact natural sciences a certain measure of security which, without mathematics, they could not attain.

The Warwick Dials
Colin Foster

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