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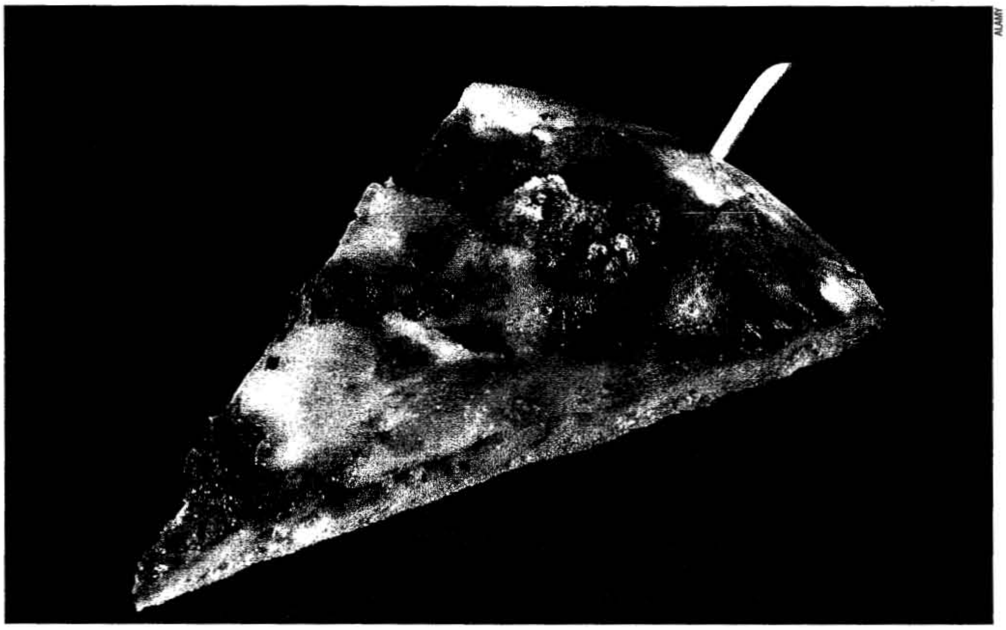

Economies of scale

Getting your head around scale can be difficult. But, take a pizza and some hungry children and they'll soon work out if their shares don't add up. **Colin Foster** reports

Our perceptions of scale can be wildly out. A recent Year 10 lesson began with the scenario where we ordered a 20-inch pizza, but two 10-inch pizzas arrived instead because they had run out. Was that OK? The class thought so, until we drew a picture ending up with the two 10-inch substitutes sitting neatly inside the much larger 20-inch pizza.

This produced reactions such as "No way!" After realising that changing the positions of the smaller pizzas did not help, it was accepted that two 10-inch pizzas are a lot less than a 20-inch pizza. One enterprising pupil commented: "You could actually do that. Most people wouldn't realise and you could make a fortune."

Trying it with a square pizza made it clear that four 10-inch square pizzas were equivalent to one 20-inch pizza. Why four? There was a lot of confusion: "4x10=20. That's stupid, Sir." The savvy pupil decided it would be a good plan to offer customers three 10-inch pizzas, "as a gesture of goodwill", to make them happy while being cheated. Did the four come from 2+2 or 2x2? We looked at square centimetres and square metres, seeing there must be 100 rows of 100 square centimetres in a square metre (10,000) and 100x100x100 cubic centimetres (one million) in a cubic metre.

This led to the idea of an "area scale factor" being the square of the "linear scale factor" and the "volume scale factor" being the cube of it. These are hard concepts to understand. So doubling all the lengths of something gives it four times as much area and eight times as much volume.

We imagined an enlarging machine, which doubled all your lengths, and realised that you would come out weighing eight times as much, but your feet would cover only four times the area. Would your bones break? And how hot might you get, with less surface area to dissipate body heat? The scale factor approach connected information into something that, though still a bit counter-intuitive, was beginning to make sense ■

Scale: useful cross-curricular resources

Books *Gulliver's Travels* by Jonathan Swift and *Alice's Adventures in Wonderland* by Lewis Carroll

Films *The Incredible Shrinking Man*, *Fantastic Voyage* and *Honey, I Shrunk the Kids*

Websites

www.rain.org/~mkummel/stumpers/10oct03a.html

www.auburn.edu/academic/classes/zy/0301/Topic4/Topic4.html

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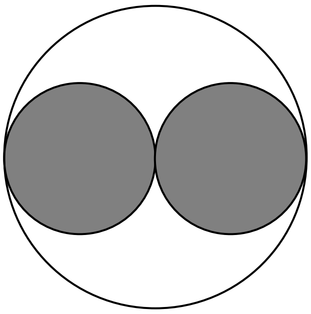
Twice As Much?
Colin Foster

Many people's perceptions of scale can be wildly out. My favourite example comes from Laurel and Hardy ('The Laurel-Hardy Murder Case', 1930), when Stan has just discovered that he may have inherited $ 3 m. He asks Ollie, "Say, is that as much as a thousand?" With great confidence, Ollie replies, "Man alive! It's *twice* as much!" Twice or three-thousand-times: does it matter?

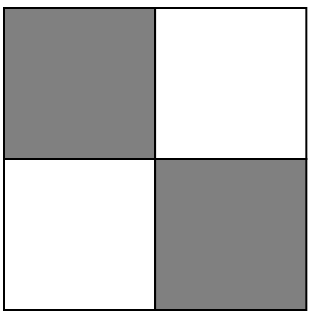
A recent Year 10 lesson began with the classic scenario in which you and your friends order a 20-inch pizza, but when the delivery person turns up at the door they say that they're sorry, they've run out of 20-inch pizzas, but here are two 10-inch pizzas instead. Is that OK?

When presented with this situation, everyone in the class felt that it would be fine; one person reckoned it would be slightly better, as it might be more convenient for sharing, but other than that it would be pretty much the same. In order to disturb this comfortable consensus (working on the principle 'Disturb the comfortable and comfort the disturbed'), I suggested we draw a picture, ending up with something like this:



Staring at this for a moment produced reactions such as "Woah!", "No way!", "Can't be!". After realising that changing the positions of the 10-inch pizzas wouldn't solve the problem, it was accepted that two 10-inch pizzas are a lot less than a 20-inch pizza, or, as one pupil more succinctly put it, "Ten plus ten *doesn't* make twenty!" One enterprising pupil, with a faraway look in his eye, said "You could actually really do that and most people wouldn't realise and you could make a *fortune*!" At last something relevant and useful coming out of a maths lesson!

There was a suggestion that this was a 'weird thing about circles', but trying it with square pizza led to the same result, and also made it clear that *four* ten-inch square pizzas would be equivalent to one 20-inch pizza:



Why four? There was a lot of confusion at this point: “Four times ten equals twenty! That’s stupid!” (The savvy pupil already mentioned now decided that the best strategy would be to offer customers *three* 10-inch pizzas, ‘as a gesture of goodwill’, alongside the apology, so that the customer would feel they were getting a *really* good deal, while in fact still being significantly cheated! What cunning! – a bright future with some unscrupulous financial institution surely beckons!)

There was some uncertainty whether the four came from 2 + 2 or 2 × 2, so we looked at square centimetres and square metres, seeing by a sketch that there must be 100 rows of 100 square centimetres in a square metre (hence 10 000 of them), and, subsequently, that there are 100 × 100 × 100 cubic centimetres (a million) in a cubic metre. This led to the idea of an ‘area scale factor’ being the square of the ‘linear scale factor’, and the ‘volume scale factor’ being the cube of it, generally pretty hard concepts to come to terms with. So doubling all the lengths of something gives it four times as much area and eight times as much volume.

We then considered Poincaré’s classic philosophical conundrum: If, while you were asleep tonight, everything (and I mean *everything*) doubled in distance, would you be able to tell when you woke up? Initially everyone thought yes, obviously, but on realising that ‘everything’ included the sun, and our eyes and hands, and the air molecules and our rulers, and so on, the consensus became that it would be impossible. (It is necessary to make some interesting assumptions about matter ‘multiplying itself’ to fill up the space created, and physical constants adjusting themselves, and so on, which is why Poincare’s contemporaries apparently couldn’t agree on the answer, and it’s now often taken as an example of a meaningless or inherently unverifiable question.)

Someone mentioned some facts they knew from biology – for example, that if you throw a caterpillar out of an upstairs window, it just crawls away, whereas if you did it to me I wouldn’t! And that if you enlarged a fly to the size of an elephant, it’s legs would snap under its weight, even assuming the size of the legs increased in proportion. We imagined climbing into an ‘enlarging machine’, which made all your lengths twice as big and realised that you would come out weighing *eight* times as much, but your feet would cover only *four* times the area, so the pressure on your legs would be *twice* as much – would your bones break? (Presumably not, actually, with just twice the pressure, since you can pick up somebody of your own weight without collapsing.) And you would get rather hot, having relatively less surface area to dissipate your body heat, and so on. Although they knew some of this from biology, the scale factor approach connected a lot of isolated bits of information into something that, though still a bit obscure and counter-intuitive, was beginning to make a bit of sense.

There are obvious cross-curricular opportunities, as well as with science, for some creative writing and drama.

Books
On Being the Right Size (1928, J B S Haldane)
Gulliver’s Travels (1726, Jonathan Swift)
Alice’s Adventures in Wonderland (1865, Lewis Carroll)

Films

The Incredible Shrinking Man (1957)
Fantastic Voyage (1966)
Honey, I Shrunk The Kids (1989)

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