Lesson plan: Mathematics KS3

A FITTING CHALLENGE

Making a child's toy can promote careful planning and understanding of 3D shapes, says Colin Foster...

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WHY TEACH THIS?

A child's toy provides a great opportunity for careful thinking about 3D shapes and their nets.

KEY CURRICULUM LINKS

+ Use language and properties precisely to analyse 2-D and 3-D shapes.

+ Use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D.

Shape sorter toys will be familiar to most students – particularly if they have a younger sibling – and designing and making one can be a good challenge for those in Key Stage 3. It requires a lot of careful thought, particularly if we want to avoid any shapes fitting through the 'wrong' hole.

Students will have to decide which shapes to include (they can go for easy or hard options), how big to make them, and how to draw out the required nets accurately. An additional constraint can be that all of the shapes should be able to fit inside the finished box at the same time.

Q Can you make a shape sorter, with at least four different shapes and holes, so that each shape will go through just one of the holes?

STARTER ACTIVITY

Ask students what they think is the point of a shape sorter. Who uses it, and what for? Establish that the toy helps young children to match a 3D shape with a hole that's the same as its cross-section. (We don't want a cylinder to go through the square hole, for instance, as this would be confusing for a small child!)

Now introduce the challenge: *"In this lesson, you are going to design and make a shape sorter of your own. Can you suggest some examples of shapes that would be well suited to this purpose?"* Students might suggest a cube, a cuboid, a cylinder, different kinds of triangular prism (e.g. right-angled, equilateral), and other prisms with more faces. A sphere would be difficult to make out of cardboard, and it might be best to avoid pyramids as it can be tricky to create holes for these.

Now ask students what they think will be tricky about the task. Drawing the nets could be challenging, and making sure that each shape goes through only one hole may be difficult; if you had two different sizes of cylinder, the one with a smaller radius would certainly fit through the hole for the one with larger radius, which would be a problem. This means that you can't have more than one cylinder in your shape sorter. Students could think about whether this applies to other shapes too, and what constraints there are on what is possible.

MAIN ACTIVITY

Before getting started, it's worth noting that students may think of shapes without knowing their mathematical names. If so, invite them to describe the shape they are thinking of, or name a common object with that shape (e.g. a Toblerone packet is an equilateral triangular prism). If necessary, they can come to the board to make a sketch of it.

If you have more than one lesson available, this can become a great extended project.

The hole picture

Lay out the details of the challenge:

"Your task is to design and make a shape sorter. I would like it to have at least four different holes and shapes. Remember that each shape should go through just one of the holes. Before you start making anything, I want you to draw out your ideas accurately."

Students should bring a suitable box (e.g. a shoebox) to school before they start; its size will determine the possible dimensions of the shapes they will later create.

Have available plain, squared and isometric paper, together with pencils, erasers and long rulers. Later on, students will need some card, scissors and glue, but it may be better not to have these to hand too early on. This will help students think carefully about what they are doing, rather than rushing in to cutting and sticking.

Some students may initially be too ambitious, so a making a cuboid can be a good place to start. A non-cube cuboid is particularly interesting because students will realise that not all of the faces are the same shape. This means they must check that it will not fit through any of the holes that are designed for other shapes, even when rotated.

Joining together

Some students may find it hard to create a net for the cuboid, and you could work on this together as a whole class, with different students coming to the board (a squared background would be ideal) and drawing and making changes to what is there until everyone agrees that it is correct (drawing it out on paper and cutting it out to check if necessary).

Students may find it helpful to superimpose the cross-sections of their shapes to ensure that no shape will go through any other shape's hole. For example, a cylinder, a cube and an isosceles triangular prism could have the cross-sections shown in fig. 1 (right, top).

However, students must be careful to consider all the different orientations of each shape. For example, if the isosceles triangular prism were very narrow, like in fig. 2, it could still fit through the circular hole using the orientation shown in fig. 3.

fig. 1

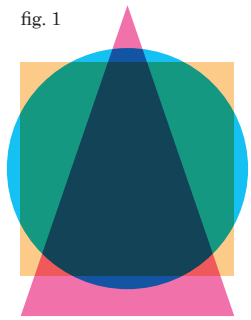


fig. 2

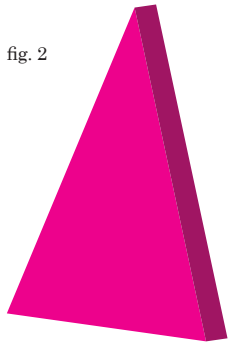
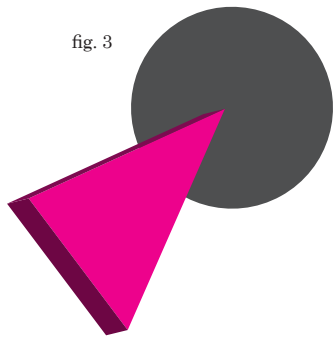


fig. 3



Sticking points

Students will gain a great deal of practice with nets as they work on this task. Remind them to include tabs on their nets before they cut them out, so they have somewhere to put the glue. (Alternatively, use sticky tape to secure the net.) If tabs are used, an approximate rule is that they should be placed on every other edge, in order, as you move around the outside of a completed net.

DISCUSSION

You could conclude the lesson (or series of lessons) with a plenary in which the students showcase the shape sorters they have produced. Ask the questions:

How did you work out the nets that you needed? How did you make sure that no shape would fit through any other shape's hole? What mistakes did you make? What problems did you encounter?

Give students time to talk about what they did. Try to encourage correct mathematical language, such as "edge", "face" and "vertex", as well as mathematical names for shapes.

GOING DEEPER

There should be plenty here to stretch all students, but anyone who really wants a challenge can attempt the following:

+ Make a shape sorter just using cuboids. Unlike cylinders, it is possible to use more than one cuboid. What is the largest number of cuboids they can include in their shape sorter?
+ Another challenge would be to make one of the shapes a *parallelepiped* – see pp 64-67, *Questions Pupils Ask*, by C. Foster (Leicester: Mathematical Association)

ONLINE EXTRA

tinyurl.com/TSsorter

This free, interactive tool is great for looking at prisms and playing around with nets.

THE AUTHOR

Colin Foster is an Assistant Professor in mathematics education in the School of Education at the University of Nottingham. He has written many books and articles for mathematics teachers (see foster77.co.uk).

Colin would like to thank Keith Proffitt, from whom he first heard of the idea of designing a shape sorter as a mathematics class project.