MATHS

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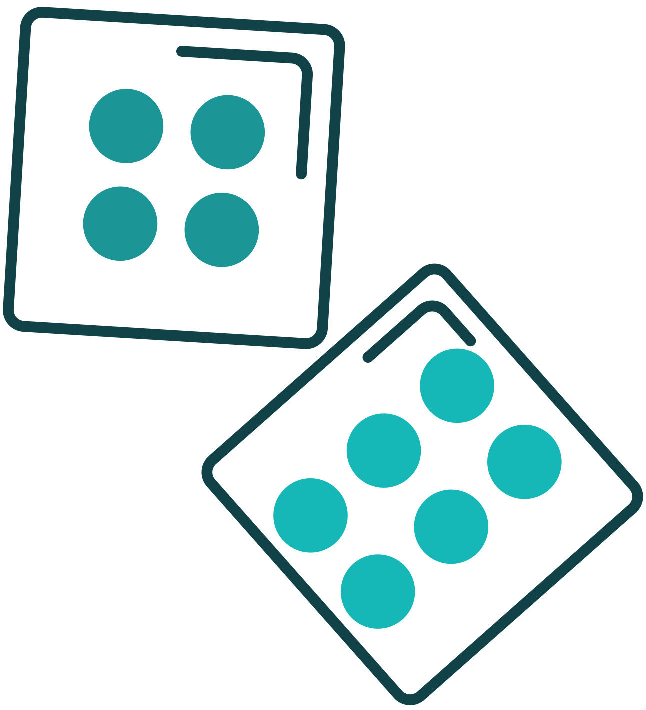
Hit Ten!

A dice game involving moving ten squares to the finishing point gives an opportunity to consider empirical probability, says Colin Foster

Why teach this?

Uncertainty is all around us in the world. Understanding probability helps us make better decisions about risk.

In many everyday scenarios, we cannot calculate a theoretical probability of something happening. Even some simple-sounding games involving dice are complicated to analyse and the easiest way to estimate the probability of a win is often to run a simulation on a computer. In this lesson, students are asked to estimate the probability of winning a game in which they throw an ordinary dice and move a counter that many spaces forwards. To win, they must 'hit ten' by landing on the tenth square from the start. This game is too complicated for students to work out the probability of winning theoretically, so they must estimate it by playing the game several times and seeing how often they win. This enables them to calculate how much money they might be willing to pay to play the game, if the prize for winning is £1.



STARTER ACTIVITY

Q. What do you think of this game?

> ***You throw an ordinary fair dice.***
> ***If you get a 5, I will give you £1.***

Students will probably be interested and say that they want to play the game. Alternatively, they might be suspicious and ask, "What's the catch?"

Q. There is a catch. You have to pay me if you want to play this game! Would you play the game? Why or why not? And if you would, how much would you be willing to pay to play? Why?

Give students a few minutes to discuss these questions in pairs and then ask for comments. You could first ask students to raise their hands if they would be willing to play the game. Then ask those unwilling why not. It may be that they disapprove of gambling or they suspect that you will cheat in some way or they think that they will lose more money than they will win, or that they simply do not have any money.

Q. Would you be willing to play the game if you had to pay 1 pence? What about 2 pence? What is the most you would pay to play this game? Would you pay 50 pence to play the game?

There might be quite a range of opinions about how much money it is reasonable to pay to play this game. You may not be able to resolve this discussion at this stage, but these questions should begin

to help students to think about probability and the likelihood of winning.
You would certainly be likely to make money playing this game repeatedly if you only had to pay 1 pence per go! But 50 pence per go probably feels too expensive, since you do not have 1 in 2 chance of winning.

In fact, the probability of obtaining a 5 on one throw is $\frac{1}{6}$, so if you played the game a large number of times, on average you would expect to win £$\frac{1}{6}$, or about 17 pence, per go. So you might be willing to play the game for an amount of money less than this, such as 15 pence, perhaps. If you paid 20 pence per go, in the long run you would lose money.

MAIN ACTIVITY

Give out the resource sheet “Hit Ten”, which is available at **tinyurl/hitten**; one sheet per pair of students (if you want to save photocopying, students can easily draw out the game themselves instead on rough paper). Each pair will also need an ordinary dice and a counter (they could use an eraser or a bit of scrunched-up paper or some other handy object).

Q. We are going to investigate a game called “Hit Ten!” In this game, you win if your counter lands on the square that says “TEN”. You have to land exactly on that square – if you overshoot it, you lose! Here are the rules:

1. Place a counter on “START”.
2. Throw an ordinary dice.
3. Move to the right that many spaces.
4. Repeat until you either hit “TEN” or go off the end.

Q. Try this game a few times. How likely do you think you are to win? Why?
Give students some time to try the game a few times and see if they win or not. Encourage them to talk about how likely they think they are to win. They might think that the answer has something to do with 10, so they might guess $\frac{1}{10}$.

Q. Suppose that if you win you get £1. I want you to decide how much you would be prepared to pay to play this game? Do you think this is a better game than the one we started the lesson with? Or worse? Why?
This is a hard game to analyse on paper, because there are many possible ways of landing on the “WIN” square. You can’t win in one throw, because the maximum score you can get in one throw is 6, but you could get there in two throws (4, 6 or 5, 5 or 6, 4) – or in, say, 10 throws (1, 1, 1, 1, 1, 1, 1, 1, 1, 1). But working out all the possible ways for three, four, five, six, seven, eight or nine throws would be quite a lot of work!

Q. How could we estimate the probability of winning?
One way is “empirically” – this means that you try the game many times and see how often you win. Encourage students to try this and keep a record of whether they win or not each time. How many times do they feel that they need to play the game before they are reasonably sure how likely you are to win?

START	1	2	3	4	5	6	7	8	9	TEN

DISCUSSION

Q. How many times did you win and lose at this game? What about other people? Did you get similar/different results? Is it a game you would pay to play? Why or why not? If you would play, how much would you pay to play?

The calculated probability of winning the game turns out to be $\frac{17\,492\,167}{60\,466\,176}$, which is about 0.29, so students should find that they win about 3 times out of 10. This means that in the long run, with a winning prize of £1, you could expect to make about 29 pence per go, on average. So you should expect to make a profit in the long run if you paid about 25 pence per go, but not if you paid as much as 35 pence per go.

An interesting (but advanced) way to get an estimate of the probability is to realise that the average score when you throw a dice once is 3.5 (you work this out by doing $\frac{1+2+3+4+5+6}{6}$). This means that in the long run you will land on one in every 3.5 of the squares. So the probability of landing on any specific (very distant) square will be $\frac{1}{3.5} = \frac{2}{7} = 0.285714$. For our 10-square game, this is quite an accurate estimate.

STRETCH THEM FURTHER

There are other enjoyable dice-based games at nrich.maths.org/6605 which, in addition to raising questions about probability, provide opportunities to develop understanding of place value and practise mental calculation and estimation.

ADDITIONAL RESOURCES

For the details of the analysis of this game, see Foster, C. (2017). Reaching the 100th square. ***Mathematics in School***, 46(3), 32–34. A good follow-on to this lesson might be to analyse the NRICH "Nice or Nasty" games described at nrich.maths.org/6605.