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Lesson plan: Mathematics KS4

JUST FOUR NUMBERS

Devising ways to make different numbers helps students make sense of mathematical operations

WHY TEACH THIS?

Combining four numbers in a multitude of ways shows the creative power of mathematics and allows students to develop their facility with priority of operations.

KEY CURRICULUM LINKS

+ Consolidate students' numerical and mathematical capability from Key Stage 3 and extend their understanding of the number system to include powers, roots (and fractional indices)
+ Calculate exactly with fractions

In this lesson, students are given just ***four*** numbers – 1, 2, 3 and 4 – and have to make as many possible different answers as they can, using common mathematical operations. This provides an opportunity for students to think about the potential ambiguity of notation, and the ways in which writing division as a fraction and using brackets can avoid this. Students will consider the priority of operations and how to be sure that an expression has one well-defined value.

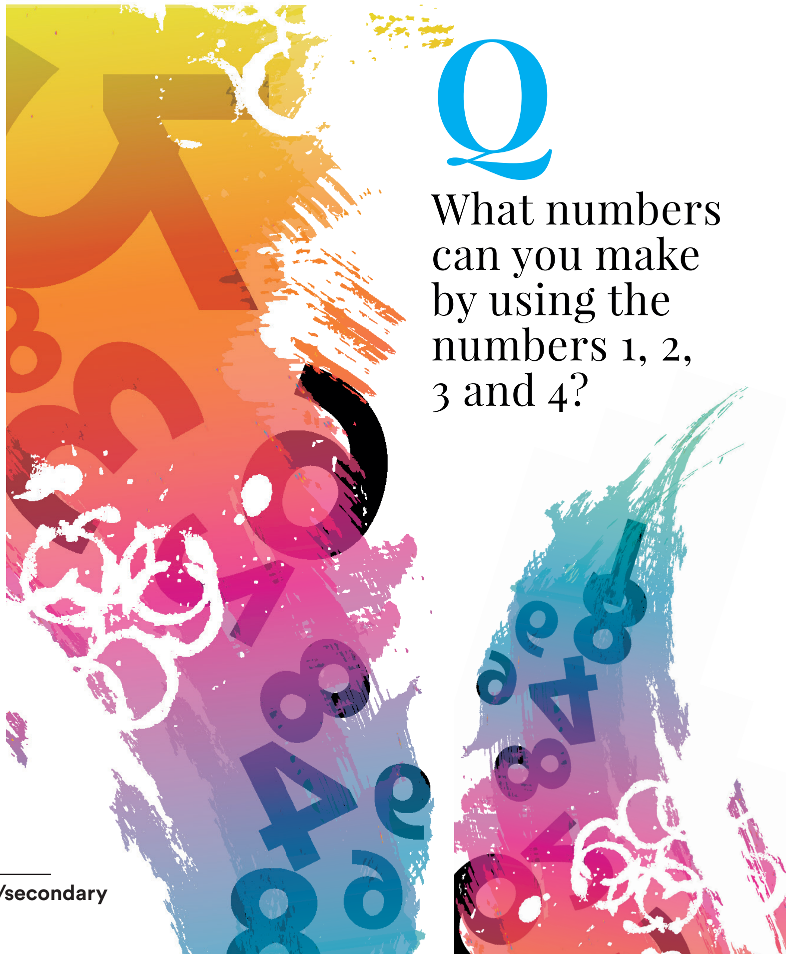
STARTER ACTIVITY

Q Can you make 24 using the numbers 1, 3, 4 and 6? You must use each number just once. No calculators, please!

This is hard, because you have to divide by a fraction. A hint could be to try $\frac{6}{\text{something}}$

Since $\frac{6}{24} = \frac{1}{4}$, the "something" must be $\frac{1}{4}$. Making $\frac{1}{4}$ using the remaining digits is much easier, giving $24 = \frac{6}{1-\frac{3}{4}}$

A similar approach can be taken to make 24 using the numbers 1, 4, 5 and 6 (again, using each number just once), so this time $24 = \frac{6}{\frac{5}{4}-1}$



Q

What numbers can you make by using the numbers 1, 2, 3 and 4?

MAIN ACTIVITY

Even if students have not managed to make 24, they will have gained some experience in playing about with the numbers and operations and thinking about how to write their answers. Students might have written things like 1-¾=¼ ÷ 6 = 24 , which contains some of the right thinking, but is not correct, because the equals signs do not denote equality, and the division is written the wrong way round. Using fraction notation, rather than the ÷ sign, may be helpful when discussing this.

Q Now, you have the numbers 1, 2, 3 and 4 and you have to use each number ***just once****. You can combine them using +, –, × and ÷ (and brackets), with as many or as few operations as you like. I want you to see how many* ***positive integers*** *you can make.*

1 = (4 − 3) × (2 − 1)	**13** = 3 × (1 + 2) + 4
2 = 1 + 2 + 3 − 4	**14** = 1 × 2 × (3 + 4)
3 = $\frac{2+4}{3-1}$	**15** = 3 × (4 − 1 + 2)
4 = 4 − 1 − 2 + 3	**16** = 2 × (1 + 3 + 4)
5 = (1 + 4) × (3 − 2)	**17** = 3 × (1 + 4) + 2
6 = 1 − 2 + 3 + 4	**18** = 1 × 3 × (2 + 4)
7 = 3 × (2 − 1) + 4	**19** = 3 × (2 + 4) + 1
8 = 4 − 1 + 2 + 3	**20** = 1 × (2 + 3) × 4
9 = 2 × 3 + 4 − 1	**21** = 3 × (1 + 2 + 4)
10 = 1 + 2 + 3 + 4	**22** = 2 × (3 × 4 − 1)
11 = 2 × 3 + 4 + 1	**23** = 2 × 3 × 4 − 1
12 = 2 × (4 − 1 + 3)	**24** = 1 × 2 × 3 × 4

Students could begin by seeing how many of the integers from 1 to 24 they can make. In fact, all of these are possible, and one way of making each is shown opposite/above.

Students may be unsure whether things like 34 – 12 = 22 are allowed, and it is probably better here **not** to allow **concatenation** (gluing) of digits.

You could have the numbers 1 to 24 written on the board, with space beside each, and students come up to the board and write up their solutions as they discover them. This allows other students to check and challenge, and may provide examples for others to build on. Alternatively, students can work alone or in pairs and then share their ideas later on.

Some students may try the opposite approach, starting with the numbers 1 to 4 and putting them together in different ways and seeing what numbers they make. This can be a faster way to hit a few numbers at the start, but as time goes on, and most numbers 1 to 24 have been made, it becomes a less efficient way for finding particular missing ones.

DISCUSSION

You could conclude the lesson with a plenary in which the students talk about the numbers that they created and how they did it.

Q Which numbers did you find it easy to make? Why? Which were hard? Are there any numbers that you ***couldn't*** *make? How did you try to do it? What worked well? Did anything surprise you?*

Some students will disagree with other students' answers when mistakes have been made, but also where there is ambiguity over priority of operations. For example, someone might declare that 1 + 3 + 4 × 2 is 16, whereas someone else might say that it is 12. Brackets are necessary to make (1 + 3 + 4) × 2 = 16, and this can be a useful discussion point.

GOING DEEPER

Students may naturally pose and solve related questions, such as **What is the largest/smallest number that you can/cannot make?** The answers depend on the rules; for example, if factorials can be used, then there is no limit to the size of a number such as 43^{21} !!!...!!!

ADDITIONAL RESOURCES

A similar problem is 'Four Fours', where you have to make numbers using common mathematical symbols and exactly four number 4s. See **paulbourke.net/fun/4444/** and **dwheeler.com/fourfours/fourfours.pdf** for very complete sets of solutions. A solution for ***any*** positive integer n is

$$\frac{\log 4}{4^{\frac{1}{\sqrt{4}}} \log \sqrt{\sqrt{\dots\sqrt{4}}}}$$

where the number of roots taken is the desired n.

THE AUTHOR

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