# KS3 Maths Lesson Plan – Explore How Proportions Work By Mixing Paints

## Thinking proportionally helps to make sense of lots of different real-life situations

Colin Foster, Assistant Professor in mathematics education in the School of Education at the University of Nottingham

Added: 03rd May 2017

**Proportionality is one of the hardest and most important ideas in lower secondary mathematics.**

Students need to encounter different examples of situations in which proportions feature, and to gain experience at representing proportions visually in different ways.

In this lesson, students work on a task about pink paint, which is made from a mixture of red paint and white paint. How much red paint do you need to add to pink paint which is 60% red to turn it into pink paint which is 80% red?

The answer may be more than you expect! Students work on several problems relating to a situation of 'sunk costs', where paint has been mixed in the wrong proportions and has to be corrected.

They will make up their own problems for each other to solve and experiment with using different ways to represent the makeup of the different mixtures of pink paint. This lesson provides lots of opportunities to explore how proportions work and gain a better sense of what something like "60% pink" really means.

Generalisations are possible, and the lesson ends with students describing what they have found out, what problems they have created for each other, and what they have learned from all of this about proportionality.

## Starter activity

***Q. Shifa makes pink paint by mixing red paint and white paint. "60% pink paint" means that 60% of the mixture***

***is red paint and the other 40% is white paint. How much red and white paint does Shifa need to use to make 5 litres of 60% pink?***

This starter question should assess how well students have understood the scenario and whether they can calculate 60% of something. They need to be able to work out that 60% of 5 litres is 3 litres, meaning that Shifa needs to mix 3 litres of red with 2 litres of white.

## Main activity

***Q. Shifa makes 5 litres of 60% pink, but then realises that she meant to make 5 litres of 80% pink instead! Oh dear! What can she do to fix it? She wants to use the smallest possible amount of extra paint to fix her mistake.***

Clearly, she could tip it all away and start all over again, but instead she wants to use some or all of the 60% pink paint that she has made.

If students have no idea what to do, you could ask:

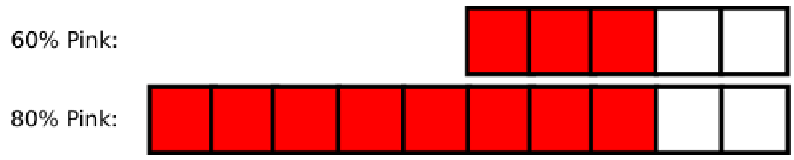
***Q. Does it need to get redder or whiter?***

Students should realise that 80% pink is redder than 60% pink, so Shifa is going to need to add some red paint. (She can't remove white paint, as it is all mixed in!) But how much? It should be clear that there is no point adding white paint – that is going in the wrong direction, and will just increase the amount of red paint she will need to add later – so Shifa just needs to add red paint.

This is quite a tricky problem. Give students some time to think about it and let them write down and talk about their ideas. You might need to emphasise that it is not possible to separate out the red and white paint from the pink mixture – it is all completely stirred in! If students are really stuck you could suggest that they start by working out how much red paint and white paint is contained in the paint that Shifa has, and in the paint that Shifa wants.

One way to think about it is to treat the white paint in the paint that Shifa has as fixed. We know that 5 litres of 60% pink paint contains 2 litres of white. So this white paint must make up 20% of the 80% pink paint, meaning that there must be another 8 litres of red. We know that 3 of these 8 litres of red paint are already in the 60% pink mixture, so this means that we need to add another 5 litres of red paint. This will give us 10 litres of 80% paint.

Students will probably be a bit surprised by the large amount of red paint that needs to be added to get from 60% to 80%. They will probably have imagined that just a litre or two would have been enough. One way to see what is going on is to make a sketch like this:




So, we have ended up with 10 litres of 80% paint, whereas we only wanted 5 litres. We had to use 5 litres of red paint to fix Shifa's mistake, and that is the same amount of paint that we would have needed to use to make 5 litres of 80% paint just starting from red paint and white paint!

We could have just mixed 4 litres of red with 1 litre of white and got what we needed without using the 60% paint that Shifa made at all!

***Q. Can we do better than this? What if we don't use* all *of the 60% paint that Shifa made?***

Give students some time to experiment with this. The idea is that we tip away some of the pink paint before we start, because that means that we don't need to add as much red paint as before to turn it pink enough.

Since we ended up with twice as much 80% pink paint as we wanted, students might start by imagining tipping out half of the 60% paint, so that they have just 2.5 litres of 60% pink. This will contain 1 litre of white paint and 1.5 litres of red paint.

To make this into 80% pink paint, 4 litres of red is needed in total, meaning that a further 2.5 litres of red paint must be added. This way we end up with 5 litres of 80% pink paint, which is exactly what we need, but this time it only entailed using 2.5 more litres of red paint.

***Q. What if we only wanted 4 litres of 80% pink, rather than 5 litres of it?***

The same "half and half" approach will still work. We take 2 litres of the 60% pink and add 2 litres of red paint. The 2 litres of 60% pink contains 0.8 litres of white and the remaining 4 – 0.8 = 3.2 litres of paint is red. This means that we have 80% pink paint, as desired.

***Q. Try working out how to get other quantities of 80% pink, and how to get 90% pink and 40% pink and so on.***

Students could present their questions and solutions on poster paper, explaining for each example how they worked it out. They might want to represent portions of the paint as rectangles (as above) or in some other way. They might be able to generalise some of their methods and write a "recipe" for how to do problems like this.

## Discussion

You could conclude the lesson with a plenary in which the students talk about the questions that they posed and the solutions that they came up with.

***Q. How did you work these out? What did you learn about how to solve problems like this? Which problems were easy and which were tricky? Why? What problems did you make up? How did you solve them? Did anything surprise you?***

It is unlikely that students will generalise this far, but it may be helpful for the teacher to have a complete sense of what is going on:

Suppose you start with $a$ litres of $p\%$ paint and you want $b$ litres of $q\%$ paint, and suppose that $p<q$, which means that we are going to need to add some red paint.

If we decide to use $k$ litres of our starting $p\%$ paint (where $k \le a$), this will give us $\frac{k(100-p)}{100}$ litres of white paint, which, to minimise wasted paint, needs to be all of the white paint in the $q\%$ paint that we want to end up with.

This means that $\frac{k(100-p)}{100} = \frac{b(100-q)}{100}$, which means that we need to start by using only $k = \frac{b(100-q)}{100-p}$ litres of our starting paint.

Doing this gets us $\frac{bp(100-q)}{100-p}$ litres of red paint, and we need $\frac{bq}{100}$ litres of red paint.

So the amount of red paint that we have to add is $\frac{bq}{100} - \frac{bp(100-q)}{100-p}$, which simplifies to $\frac{b(q-p)}{100-p}$. (Note that the restriction that $k < a$ – ie we cannot use more of the starting paint than there is! – sets a limit on how large $b$ can be: $b < \frac{a(100-p)}{100-q}$.)

In the problem in the main activity, $p\%$ was 60% and $q\%$ was 80%, and because $\frac{80-60}{100-60}$ was ½ we always needed to add red paint equal to half of the amount of paint we wanted. But for other values of $p\%$ and $q\%$ this will not necessarily be the case. So there is much to explore here that will be challenging for students to explain.

## Additional resources

A nice related problem, with a video introduction, is available here at blog.mrmeyer.com.

## Stretch them further

There should be plenty here to stretch all students, but anyone in need of a greater challenge could suppose that the red paint is a bit more expensive than the white paint. This complicates the model in interesting ways!

---

**Colin Foster is an Assistant Professor in mathematics education in the School of Education at the University of Nottingham. He has written many books and articles for mathematics teachers (see foster77.co.uk).**