WHY TEACH THIS?

Our world is full of numbers, but not all of them are created equal. Some — the prime numbers — play a critical role in enabling us to generate all the other whole numbers. Learning about the prime numbers and how they work is a crucial part of understanding about number.

Sometimes students are aware of prime numbers as those whose only factors are “one and itself”. This definition can lead to ambiguity about whether 1 itself should or should not be counted as prime, but another problem is that it can seem a very arbitrary definition. Why should numbers with exactly two factors be worth giving a name to and learning about? Students may learn about factorising numbers into primes without realising why this is of any more significance than factorising numbers into, say, square numbers or even numbers or any other kind of numbers. This lesson focuses on what it is about the prime numbers that makes them special — the idea that they are the building blocks of all the whole numbers greater than 1. So breaking a number down into its prime factors is a bit like breaking down a chemical molecule into its constituent elements — you really discover its structure. Since every whole number can be factorised into a unique product of primes, the prime numbers really form the basis for generating all the whole numbers that there are!

STARTER ACTIVITY

Q. Do you think that some numbers are more important than other numbers or are all numbers equally important? Why?

Students might think about this for a minute in pairs before sharing their ideas. There is no single right answer; the aim of this initial question is just for students to think about the possibility of some numbers being more important or useful than others. They might say, for example, that 12 is the most important number because it is their age, or because it has a lot of factors. They might mention telephone numbers or numbers that you see a lot in the world as being particularly important.

Q. I want to make all the whole numbers from 1 to 20 by multiplying numbers together. What would be a good set of starting numbers to use?

As students suggest possibilities you can introduce the idea of the smallest possible set of starting numbers that will do it. Again, this could be worth thinking about in pairs before sharing. Students might say that 2s are very useful because you can make all the even numbers, but that involves adding, and here we are restricted to multiplying only. So if you have a box of 2s then you can only make the powers of 2, not all the even numbers. So you need 2s and 3s, for instance, but you don’t need 4s or 6s, because you can make those out of 2s and 3s. Likewise, students may think that 1s are useful, whereas in fact 1 is useful only for making 1 and doesn’t contribute to making any other number, because multiplying by 1 has no effect.

The smallest set of numbers that will do it are: 1, 2, 3, 5, 7, 11, 13, 17 and 19; the first 8 prime numbers, together with 1. Students may use the language of prime numbers or they might not at this stage.
If no one has any idea, you could suggest that they might have seen it around the science classrooms.

Q. What is special about the columns of just these 100 or so elements, just like every English word is made up of combinations of the 26 letters of the alphabet?

If you have written down the product of all the primes up to and including 200, you would write down a very big number, but would that number be prime? None of your 1000 primes would go into it, because they all leave a remainder of 1. So either this big number is a prime or it has a prime factor that is bigger than any of your 1000 primes. Either way, the 1000 primes on your list can’t be all of the primes. This argument would work no matter how many primes you had written down on your list. So the primes must go on forever.

Students could begin working out which primes they need to multiply together to make a number, like 12 = 2 × 3 or 2 = 2. They will find that each whole number greater than 1 has exactly one way of being expressed as a product of primes (ignoring the order in which you write the product). This is called unique factorisation.

Q. What patterns do you notice in the table?

There are lots of things that students can point out. Students could be asked to make a poster to illustrate what they find.

For example, some columns never contain any primes after the top row. Why should this be?

Q. Do you think the primes numbers go on for ever, or do you think they eventually stop, so there is a last prime number?

This can be a nice opportunity for a simple proof by contradiction, although you don’t necessarily need to use that language. Suppose there are exactly 1000 primes and no more, and you write them down on a piece of paper. Imagine multiplying them all together and adding 1. It would make a very big number, but would that number be prime? None of your 1000 primes would go into it, because they all leave a remainder of 1. So either this big number is a prime or it has a prime factor that is bigger than any of your 1000 primes. Either way, the 1000 primes on your list can’t be all of the primes. This argument would work no matter how many primes you had written down on your list. So the primes must go on forever.

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