Lesson plan: MATHS KS4

SINE LANGUAGE

Get them exploring this important method for solving triangles, says **Colin Foster**

WHY TEACH THIS?

The sine rule is a powerful way to capture the relationships between the side lengths and angles in a triangle.

KEY CURRICULUM LINKS

+ know and apply the sine rule

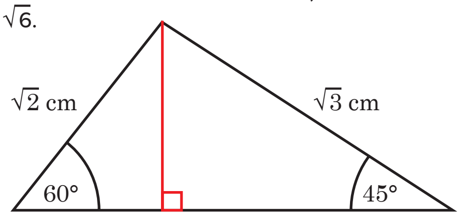
In this lesson, students will explore how the sine rule allows possible triangles to be solved, given combinations of two angles and a side or two sides and an angle that is not in between. They are asked to invent angles and side lengths that will lead to (i) a unique solution, (ii) two possible solutions (the 'ambiguous case'), and (iii) no solutions (no such triangle). Experimenting with different values helps students to see when each particular scenario will arise.

Q How can we use the sine rule to find a missing side or angle in a triangle?

STARTER ACTIVITY

Q *Find **two ways** of calculating the length of the red line segment in this figure.*

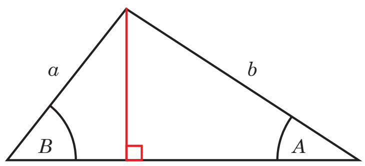
Students can use the sine ratio in either the left-hand right-angled triangle or the right-hand right-angled triangle. The red length is either $\sqrt{2}\sin 60°$ or $\sqrt{3}\sin 45°$, both of which are equal to $\frac{1}{2}\sqrt{6}$.



(Students must make sure that their calculators are set to 'degrees' mode throughout this lesson!)

MAIN ACTIVITY

Q Now we'll do exactly the same thing, but with letters.



This time the red line is equal to both $a \sin B$ and $b \sin A$, so

$a \sin B = b \sin A$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Q Now choose values for *A*, *B* and *a*, and sketch your triangle.

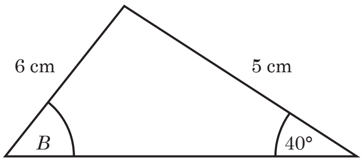
Then use the equation to work out *b*. Do three different triangles and then check your neighbour's three calculations. Always check that the bigger side is opposite the bigger angle – otherwise something is wrong!

Starting from $a \sin B = b \sin A$, or by inverting both sides of our sine rule equation, we can get

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

which is more useful when we want to work out a missing ***angle*** rather than a missing side.

Q Let's try to work out angle *B*.



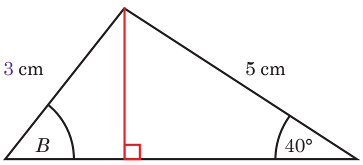
$$\frac{\sin 40°}{6} = \frac{\sin B}{5}$$

which gives $B = 32.4°$, correct to 1 decimal place.

Students must be careful not to round $\sin B$ prematurely, but to use the 'ANS' button on the calculator so as to keep all the digits for the $\sin^{-1}$ step.

[Note that although, as drawn here, B looks bigger than A, students must trust the numbers and not the visual impression. $A > B$, since $a > b$.]

Q What happens in this case?



Students will find that, this time,

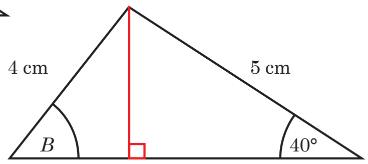
$$\frac{\sin 40°}{3} = \frac{\sin B}{5}$$

giving a value of $\sin B$ that is ***more than 1***, which is impossible (an error on the calculator when doing the $\sin^{-1}$ step, since the sin function is always less than or equal to 1.)

Q Why does this happen?

If students stare at the triangle, they may suspect that side a is now ***too short*** to meet the base of the triangle. (They could try to draw the triangle with ruler, protractor and compasses.) We can check this by calculating the length of the red line, which must be 5 sin 40° = 3.2 cm (correct to 1 d.p.). Side a, as a slanting side, has to be longer than the perpendicular height, 3.2 cm, and it isn't. So, in this case, there is ***no such triangle***, which is why the algebra led to an impossible value. That's good!

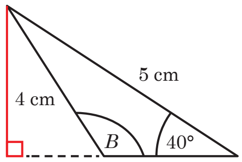
Q Finally, let's try one more case.



$$\frac{\sin 40°}{4} = \frac{\sin B}{5}$$

which gives $B = 53.5°$, correct to 1 decimal place. However, because 4 cm < 5 cm, there are actually ***two possible triangles*** here.

The other possibility has B as an *obtuse* angle.



$\sin B$ will have the same value either way, but here B will be 180 – 53.5° = 126.5° (correct to 1 d.p.). There will always be an ***ambiguous case*** in this arrangement whenever the a side is shorter than the b side, and can swing round towards the b side and make an obtuse angle.

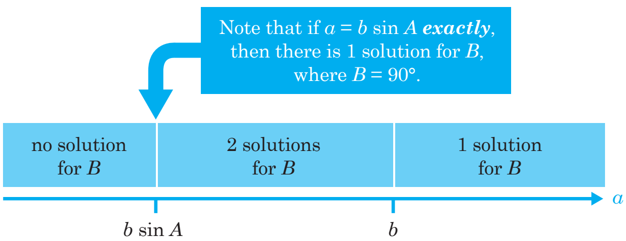
Q I want you to create three more examples, each with 2 lengths and 1 angle of your choosing, that recreate these three possibilities when you try to calculate the missing angle. Draw your triangles and show the calculations that go with them.

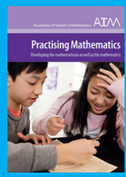
DISCUSSION

*Q What examples did you create? Do you have an example which turns out **not** to be a triangle? Do you have an example which has **exactly one solution**? Do you have an example that has **two solutions**?*

Share students' creations and check together that they behave as intended.

In summary, for A < 90°,





Practising Mathematics – Developing the mathematician as well as the mathematics (by Dave Hewitt and Tom Francome)

This fantastic resource from the Association of Teachers of Mathematics is full of ideas and activities for practising the content of the secondary mathematics curriculum, including trigonometry and the sine rule, while developing reasoning and problem solving skills. **www.atm.org.uk/shop/act107pk**

ADDITIONAL RESOURCES

There is a nice, interactive applet at **geogebra.org/m/AvvWw2v9**, which allows students to vary *A*, *a* or *b* and see what happens to the triangle.

GOING DEEPER

Confident students could find out about the relationship between the sine rule and the triangle's **circumcircle**.

THE AUTHOR

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