Lesson plan: MATHS KS4

SPOTTING SEQUENCES

The most obvious patterns are not always the ones we should be looking for, says **Colin Foster**

DOWNLOAD

a FREE lesson on number patterns for KS3, by Colin Foster, at

teachwire.net/ks3patterns

WHY TEACH THIS?

Attending to deep structure, rather than superficial patterns, is essential for tackling difficult mathematical problems

KEY CURRICULUM LINKS

+ make and test conjectures about the generalisations that underlie patterns and relationships;
+ look for proofs or counter-examples

In this lesson, students will explore a situation in which a simple pattern of numbers develops. However, this simple pattern does not continue beyond the 5th term, and it turns out that something more complicated is going on. The purpose of the lesson is to encourage students to avoid making quick assumptions about number sequences and instead to look carefully at the details, exploring the mathematical structure that underlies the numbers.

Q What might the next number in a sequence be?

STARTER ACTIVITY

Q *What number comes next?*

2, 3, 6, …

Students might think about patterns in the differences:

2, 3, 6, 11, …

+1 +3 +5

Or they might think about multiplying the previous two numbers:

2, 3, 6, 18, …

Or multiplying ***all*** the previous numbers:

2, 3, 6, 36, …

Or something more complicated:

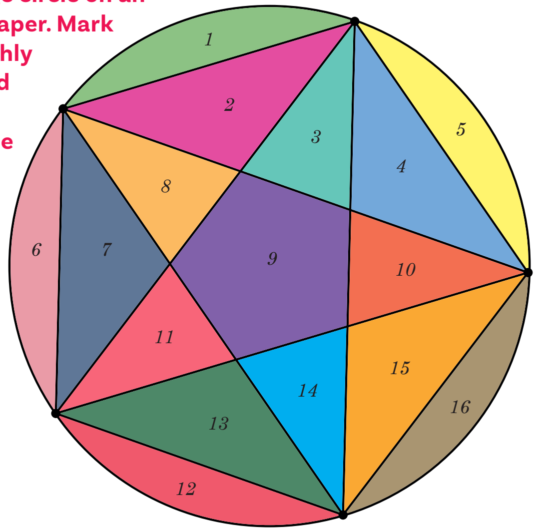
2, 3, 6, 15, …

×1.5 ×2 ×2.5

The point of this task is to be creative and to realise that there are multiple possibilities. There is not just one right answer. ***Any*** number could come next, provided that you have a convincing reason for it.

MAIN ACTIVITY

Q Draw a large circle on an A4 piece of paper. Mark 5 points roughly evenly spaced around the circumference of the circle. Join every point to every other point. How many regions (separate areas) are there inside the circle?



Students will have to be quite careful with their drawing to produce a diagram like this, and also be accurate with their counting. A good check is to make sure that every dot has the same number of lines coming out from it. They should find 16 regions altogether.

Q Explore what happens with other numbers of dots around the circumference. Check carefully and don't make hasty assumptions!

You may wish to suggest that students set up a table like this, and you might want to divide up the work, with different students working on different numbers of dots:

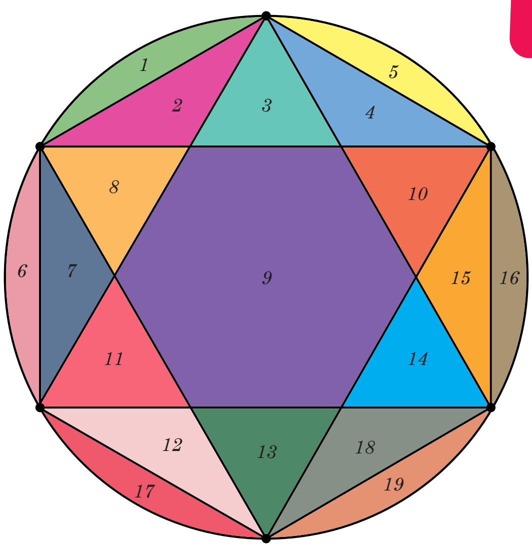
no. of dots	no. of regions
1	
2	
3	
4	
5	16
6	
7	

There is a surprise in this task. Because the sequence begins 1, 2, 4, 8, 16, …, students are likely to think that the pattern is simply 'doubling', according to the formula 2^{n-1}, where n is the number of dots. Although this works for $1 \le n \le 5$, it fails after that, so, if students write 32 for $n = 6$ and 64 for $n = 7$, you could ask: *"Did you really count all the regions or is that a conjecture?"*

DISCUSSION

Q *What numbers did you get?*

There may be considerable disbelief that the answer for $n = 6$ is ***not*** in fact 32 and is actually 31. A really accurate drawing such as the one on the right may finally convince them:



It is good if students feel shocked and disturbed by this – it ***is*** counterintuitive. In the starter, we just gave the first 3 numbers of a sequence, so it is perhaps less surprising that there are multiple possibilities for the 4th term. But here we are talking about the **6th** term of what looks like a very simple well-known sequence.

The correct values are:

no. of dots	no. of regions
1	1
2	2
3	4
4	8
5	16
6	**31**
7	**57**

Instead of 2^{n-1}, the formula is actually $\frac{1}{24}(n^4 - 6n^3 + 23n^2 - 18n + 24)$. It is quite hard to derive this formula. One way to think about it is that the total number of regions must be 1 + number of chords + number of intersections inside the circle. (Students will need to ponder this to see that it is true.) To work out these values requires some awareness of 'combinations', and you might just want to suggest the relevant ideas rather than go into any detail. There must be ${}^nC_2$ chords, because each chord has a dot at each end, and there must be ${}^nC_4$ intersections, because each one is defined by 4 points on the circle. This gives the formula $1 + {}^nC_2 + {}^nC_4$, which when expanded turns out to be $\frac{1}{24}(n^4 - 6n^3 + 23n^2 - 18n + 24)$. Even if students don't follow this explanation, they should take away the idea from this lesson that simple assumptions can be dangerous and patterns don't always continue in the simplest, most obvious way. It is always more important to probe what the structure of the mathematics is than just assume that a simple pattern will continue.

ADDITIONAL RESOURCES

There is a nice, interactive 'mystic rose' drawing tool at nrich.maths.org/6703. There is also more information about the task used in this lesson at: Cornelius, M. L. (1975). Variations on a Geometric Progression. *Mathematics in School*, *4*(3), 32-32.

GOING DEEPER

Confident students could explore the number of factors of *n*! (*n* factorial). This begins as the same simple-looking sequence and also 'goes wrong' at the 6th term, but in a different way: 1, 2, 4, 8, 16, 30, 60, 96, 160, 270, … The values are fairly easy to work out if you write the factorials in prime factorised form.

ABOUT OUR EXPERT

Colin Foster is a Reader in Mathematics Education at the Mathematics Education Centre at Loughborough University. He has written many books and articles for mathematics teachers. His website is **www.foster77.co.uk**, and on Twitter he is **@colinfoster77**.