SUM FRACTIONS

THEY'VE BEEN TAUGHT THE TECHNIQUE A DOZEN TIMES OR MORE… BUT THROUGH EMBEDDING IT WITHIN A RICHER PROBLEM, 'ADDING AND SUBTRACTING FRACTIONS' MIGHT FINALLY MAKE SENSE TO STUDENTS, SAYS COLIN FOSTER…

WHY TEACH THIS?

Developing fluency with a particular mathematical technique to the point where students no longer need to think about the details can give them a powerful sense of mastery. This enables them to tackle more demanding mathematical problems that rely on that process and contributes to a wider confidence in the subject.

Adding and subtracting fractions is a procedure which students often find it very difficult to master, despite sometimes having been taught it every year from upper primary school to the sixth form. They'll do it for a few lessons but then quickly forget it again for another year – particularly in this age of calculators and computers. When revisiting this topic at Key Stage 4, therefore, it can bring up powerful memories of failure, so it is important to address the area without it feeling like an exact repetition of what they have done before. One way to make the practice of a technique less tedious than a page of exercises is to embed it in a bigger problem which students are trying to solve. Having their attention on a grander purpose helps them to internalise the method to the point where they don't have to think about it any more, which is always the goal of mathematical fluency. A problem-based approach also means that the answers obtained have some significance for the wider problem, so students are more inclined to check their answers and notice if any of them doesn't seem to be about the right size.

STARTER ACTIVITY

Q. What can you say about these six fractions?

$$\frac{1}{6} \quad \frac{1}{25} \quad \frac{3}{5} \quad \frac{3}{20} \quad \frac{4}{15} \quad \frac{5}{8}$$

Students might note that they are all different, that they are all less than 1, that they are all positive, that they are all expressed in their simplest terms, that four are less than a half and two are greater than a half, that they are not in order of size, and so on. Encourage students to say as many things as they can think of. Questions like this are a good way to encourage students to be ***mathematically observant.***

Q. Which fraction do you think is the largest? Which is the smallest? Why?

This next one is hard, and it might be easier to begin by asking pairs of students:

Q. Pick two fractions and say which one is larger.

Since all of the fractions are expressed in their simplest terms (cancelled down), it is easy to see that none of them are equal. A common way to compare fractions is to make their denominators equal. Another way is to convert them to decimals. However, in some cases, neither of these methods might be needed. For example, $\frac{1}{20}$ is bigger than $\frac{1}{25}$, so $\frac{3}{20}$ will certainly be bigger than $\frac{1}{25}$. Encourage students to articulate reasoning like this and only to calculate when it becomes absolutely necessary.

No pair of students by themselves may be able to find the order of the entire list, but this may be possible as a whole class if the work of ordering pairs of fractions is divided up. The order is $\frac{1}{25} < \frac{3}{20} < \frac{1}{6} < \frac{4}{15} < \frac{3}{5} < \frac{5}{8}$.

Q. Tell me another fraction that is equal to $\frac{3}{5}$. And another, and another. Explain how you know that they are all equal to $\frac{3}{5}$.

Students could write their fractions on mini-whiteboards. They will probably list equivalent fractions such as $\frac{3}{5}$, $\frac{6}{10}$, $\frac{300}{500}$, etc.

Then ask them to do the same thing with $\frac{5}{8}$.

Q. How would you add $\frac{3}{5}$ and $\frac{5}{8}$ without a calculator?

Here the equivalent fractions $\frac{24}{40}$ and $\frac{25}{40}$, with common denominators, are particularly useful. $\frac{24}{40} + \frac{15}{40} = \frac{49}{40}$. The answer could be written as $1\frac{9}{40}$ or it could be left as a top-heavy (improper) fraction.

Q. The answer to $\frac{3}{5} + \frac{5}{8}$ is a little bit more than 1. Is there any way that you could have predicted that the answer was going to be more than 1 without working it out exactly?

Since 3 is more than half of 5, and 5 is more than half of 8, both fractions are more than $\frac{1}{2}$, so their total must be more than 1. This sort of reasoning can be very useful for estimating the size of an answer so that mistakes can be spotted. Estimation will be important in the main activity (next page).

If students are shaky on the process of adding fractions they will get lots of practice during the lesson. It may be best to ensure that confident and less confident students are mixed up in the room, so that less confident students have some expertise nearby and more confident students have someone to explain to. This should benefit both.

MAIN ACTIVITY

Q. I want you to add together as many of the six fractions as you like to get an answer that is as near to 1 as possible. You can use each fraction only once.

Make sure that students understand that they can choose whichever of the six fractions they wish and can add just two fractions, or three fractions, or more. It is probably best for them not to use calculators, since one of the intentions is that they gain practice at carrying out the process of adding fractions themselves. However, if you want to focus on making estimates then calculators could be useful for checking.

If some students are unsure how to start, you could ask people to share ideas with the whole class about how they will begin. Alternatively, you can suggest that they sort out an approach within their small groups.

Some students will probably begin by keeping either the $\frac{3}{5}$ or the $\frac{5}{8}$ and adding on a smaller fraction. Others might start with 1 as the goal and subtract fractions from it to see what is left. Others might just choose two or three fractions at random and add them together to see what will happen. Some might try to find a lowest common denominator for all six fractions (600) and express them all with this same denominator. Estimation comes into play in judging the size of the anticipated answer so that sensible combinations of fractions are chosen.

Students will probably end up quite close to 1 fairly soon. If they obtain an answer like $\frac{11}{12}$ (from $\frac{3}{5}$ + $\frac{1}{6}$ + $\frac{3}{20}$), they may think that they are as close as possible, as their answer is "only 1 away", but because the "one" is "one twelfth" they are not really that close ($\frac{1}{12}$ is more than 8%), so they should aim to get even closer!

If students find the best possible solution, another problem they could tackle is to find the set of fractions which add up to as near to $\frac{1}{2}$ as possible. (The answer to this is $\frac{4}{15} + \frac{1}{6} + \frac{1}{25} = \frac{71}{150}$.)

SUMMARY

You could conclude the lesson with a plenary in which the students talk about what they have found out and learned. Is it possible to make 1 exactly? How do you know? How close to 1 did different groups get? What strategies did they follow?

Although the task may appear to be mainly about addition of fractions, in order to decide whose answer is closest to 1 some subtraction may be necessary. If the answer is greater than 1, then subtracting 1 is quite easy, especially if the answer is written as a mixed number. If the answer is less than 1, it is probably a bit harder to calculate the difference.

It is not possible to make 1 exactly. The five nearest sums, in order, are:

$$\frac{3}{5} + \frac{4}{15} + \frac{3}{20} = 1\frac{1}{60}$$

$$\frac{5}{8} + \frac{1}{6} + \frac{3}{20} + \frac{1}{25} = \frac{589}{600}$$

$$\frac{3}{5} + \frac{4}{15} + \frac{1}{6} = 1\frac{1}{30}$$

$$\frac{5}{8} + \frac{4}{15} + \frac{3}{20} = 1\frac{1}{24}$$

$$\frac{3}{5} + \frac{1}{6} + \frac{3}{20} + \frac{1}{25} = \frac{287}{300}$$

INFORMATION CORNER

ABOUT OUR EXPERT

Colin Foster is a Senior Research Fellow in mathematics education in the School of Education at the University of Nottingham. He has written many books and articles for mathematics teachers (see www.foster77.co.uk).

STRETCH THEM FURTHER

STUDENTS WHO WANT ADDITIONAL CHALLENGES COULD MAKE UP A SIMILAR PROBLEM USING THEIR OWN SET OF SIX FRACTIONS. WHAT CHOICES OF FRACTIONS MAKE THIS A GOOD PROBLEM AND WHAT CHOICES DON'T? THEY SHOULD TRY TO MAKE THE PROBLEM NEITHER TOO EASY NOR TOO DIFFICULT. ANOTHER PROBLEM STUDENTS COULD CREATE WOULD BE A SET OF 10 FRACTIONS IN WHICH A SUBSET ADD UP TO 1 EXACTLY. WHAT WOULD BE A GOOD CHOICE OF FRACTIONS TO USE FOR THIS PROBLEM SO THAT THE ANSWER ISN'T OBVIOUS?

ADDITIONAL RESOURCES

FOR STUDENTS WHO WANT SOMETHING VISUAL TO HELP THEIR THINKING, A FRACTION NUMBER LINE IS AVAILABLE AT **www.mathsisfun.com/numbers/fraction-number-line.html**