WHY TEACH THIS?

The world around us is full of symmetrical and almost-symmetrical objects. Even our human bodies have an approximate vertical plane of symmetry. Understanding about reflection and rotational symmetry helps us to appreciate designs and patterns and prepares students for a deeper appreciation of mathematical structure.

SYMMETRY COMBINATIONS

RATHER THAN TREATING THEM SEPARATELY, EXAMINING REFLECTION AND ROTATIONAL SYMMETRY TOGETHER HELPS STUDENTS TO UNDERSTAND THEIR SIMILARITIES AND DIFFERENCES, SAYS COLIN FOSTER...

Students are often asked to say how many lines of symmetry and what order of rotational symmetry a given shape has, and it can seem as though these two kinds of symmetry are quite separate from each other. But they are related. A shape with exactly 5 lines of symmetry *must* have order 5 rotational symmetry. But a shape with order 5 rotational symmetry doesn't necessarily have to have 5 lines of symmetry. It might have 5 lines of symmetry, or it could have none. Are there any other possibilities for a shape with order 5 rotational symmetry? What about a shape with order 6 rotational symmetry? In this lesson, students are asked to think of shapes with different numbers of lines of symmetry and different orders of rotational symmetry in order to see what is possible and what is impossible. By thinking about both kinds of symmetry together they have the opportunity to deepen and enrich their understanding and explore what features of a shape contribute to the different kinds of symmetry that it may have.

STARTER ACTIVITY

Q. Which letters of the alphabet do you think are the most symmetrical? Let's just use capital letters for now.

Whatever letters students offer, ask them to say why and to describe the kind of symmetry that the letter has. You could encourage the use of precise mathematical language, such as ***line of symmetry, order of rotational symmetry*** and ***centre of rotation***.

Students could draw their letter on the board, or if you have a projector you could type it, in a very large font size, and they could then draw on the lines of symmetry and centre of rotation. Some answers might depend on the particular font used, and in general ***sans serif*** fonts are likely to have the most symmetry. For example, in the word MATHS as written below, every letter has symmetry of some kind. M, A and T each have one vertical line of symmetry, H has one vertical and one horizontal line of symmetry and S has no lines of symmetry. H and S each have order 2 rotational symmetry, since they look exactly the same when rotated by 180° about their centres. M, A and T have no rotational symmetry, which we can call "order 1", because they look the same only when rotated through (a multiple of) 360°.

In some fonts, the H might have its horizontal bar not exactly half way up, so would have just 1 line of symmetry and order 1 rotational symmetry. Similarly, the letter K needs to be examined quite carefully to see whether it has a line of symmetry or not. It can be interesting if students comment on these sorts of details. The letter O is special, as it has infinitely many lines of symmetry (between any two lines of symmetry we can draw another one) and an infinite order of rotational symmetry, because rotation by any amount, however small, leads to no change.

If students disagree with each other about the symmetries that the letters have, carry out the reflection or rotation for them to see and compare with the original letter. By the end of this starter, students need to understand the terms ***number of lines of symmetry*** and ***order of rotational symmetry***.

+KEY RESOURCE

Mathematics in School is a flagship journal of The Mathematical Association. It's aimed at teachers of school and college pupils aged 10 to 18 years and for those working with students preparing to become teachers. Content is a balance of articles, puzzles and classroom activities that reflects the interests of the readership; it's designed to stimulate – and even amuse – otherwise hard-pressed and very busy teachers. Contributions to Mathematics in School are written by teachers of mathematics for teachers of mathematics. The editors are keen to welcome new authors so submit your favourite activity – maybe it's a short starter for a topic or a Friday afternoon puzzle session; if it works for you, it will work for other teachers too – to **mis@m-a.org.uk** or visit **www.m-a.org.uk/mathematics-in-school**

MAIN ACTIVITY

Give pairs of students a copy onto A3 paper of the possibility table shown right.

Q. Draw or write the name of some shapes that will go in each of the boxes. Do you think you will be able to find a shape to go in every box? You might want to use pencil and rubber in case you want to change your mind about what you put in one of the boxes.

Students can begin wherever they like on the possibility table. They might start by thinking of a shape (or a letter, as in the starter) and then consider where it should go. Or they might take a particular box on the table and think about what kind of shape might belong in it.

As you circulate, bear in mind that sometimes empty boxes are just boxes that the students haven't yet thought about, and sometimes they might have thought about them and think that no shape can go in them. When students say that a particular box is impossible, it may be helpful to suggest that they write "impossible" in the box, or cross it out. See if they can say ***why*** they think that no shape could possibly go in that box.

If students draw shapes in the wrong boxes they may later realise their error as they struggle to make sense of the patterns in the table, so self-correction can often take place. It may not be necessary for you to correct every mistake as soon as you see it.

Number of Lines of Symmetry \ Order of Rotational Symmetry	1	2	3	4	5	6
0						
1						
2						
3						
4						
5						
6						

A task sheet containing this possibility table is available at **www.teachsecondary.com/downloads/maths-resources**

DISCUSSION

You could conclude the lesson with a plenary in which the students talk about the shapes that they have found/invented and where they think they go in the possibility table.

*Q. Tell me one of your shapes and where it goes. Why does it go there? Tell me a box that it was hard to find a shape for. Which box did you find the hardest to fill? Did you change your mind about anything as you were working on this? Why? Are there any boxes or shapes that you are still unsure about? Are there any boxes that you don't have shapes for? Are there any boxes that you think **can't** have any shapes in? Why?*

The table below shows some examples of shapes to go in all of the possible boxes, along with the relevant symmetry groups (for the benefit of the teacher). The teacher might realise that the only finite symmetry groups in R^2 are Z_n (the cyclic group) and D_n (the dihedral group). The Z_n across the top row have rotational symmetry but no reflection symmetry, whereas the D_n down the diagonal (after D_1) have both. Students might recognise that these families of shapes extend beyond the limits of the table along the top row and down the diagonal. They might also wonder where a circle would go, with infinitely many lines of symmetry and an infinite order of rotational symmetry. (This is the orthogonal group O(2).)

Some people might prefer to discourage students from drawing Swastika-type shapes for the top row because of Nazi associations, but the symbol dates much earlier as a sacred Hindu and Buddhist sign.

Number of Lines of Symmetry \ Order of Rotational Symmetry	1	2	3	4	5	6
0	Z_1 Scalene Triangle	Z_2 Non-rectangular Non-rhombus Parallelogram (Rhomboid)	Z_3 Triskelion	Z_4 Swastika	Z_5	Z_6
1	D_1 Isosceles Triangle					
2		D_2 Oblong				
3			D_3 Equilateral Triangle			
4				D_4 Square		
5					D_5 Regular Pentagon	
6						D_6 Regular Hexagon

INFORMATION CORNER

ABOUT OUR EXPERT

Colin Foster is an Assistant Professor in mathematics education in the School of Education at the University of Nottingham. He has written many books and articles for mathematics teachers (see www.foster77.co.uk).

ADITIONAL RESOURCES

A NICE ANIMATION TO ILLUSTRATE ROTATIONAL SYMMETRY IS AVAILABLE AT **HTTP://FLASHMATHS.CO.UK/VIEWFLASH.PHP?ID=31**

STRETCH THEM FURTHER

Students could try to justify why a shape with 4 lines of symmetry *must* have order 4 rotational symmetry (even though the converse is not true). They could think about what happens if you start with a shape and reflect it twice in two perpendicular mirror lines (why must they be perpendicular?).