Lesson plan: MATHS KS3

THE TETHERED GOAT

When a goat is tethered to a shed, how much of the surrounding area can they reach? Colin Foster explains how this is a rich scenario for exploring circle area calculations...

WHY TEACH THIS?

This lesson applies students' knowledge of loci and the area of a circle to solve a problem involving a tethered goat.

KEY CURRICULUM LINKS

- Make and use connections between different parts of mathematics to solve problems
- Calculate and solve problems involving the perimeters of 2D shapes (including circles), areas of circles and composite shapes

In this lesson, students consider the parts of a field a goat can reach if it is tethered to the corner of a shed. The region of grass the goat can access depends on the length of the rope; as the rope gets longer and begins to snag on the corners of the shed, students will need to think hard in order to work out the locus and calculate the total area of accessible grass.

Q How much grass can a tethered goat reach?

STARTER ACTIVITY

Q *Imagine a goat is tethered by a rope to a post in the middle of an empty field of grass. What shape area of grass can the goat eat? Why?*

Students should realise that the locus of points in a plane that are a fixed distance from a fixed point is a circle. Since the rope is flexible, the goat will be able to eat all of the grass inside that circle.

Encourage students to use precise language and vocabulary to answer the question, and state that the radius of the circle is equal to the length of the rope, and that the centre of the circle is at the base of the post.

Q *What assumptions do we need to make to answer this question?*

Students might say many things in response to this – that the rope is not stretchy and doesn't break; the post is fixed; the ground is flat; the grass in the field is evenly distributed; the field is larger than the area the goat can reach; there is nothing else in the field, etc.

MAIN ACTIVITY

Q *Now suppose that the goat is tethered to the corner, O, of a shed that is 4 metres one way by 6 metres the other way.*

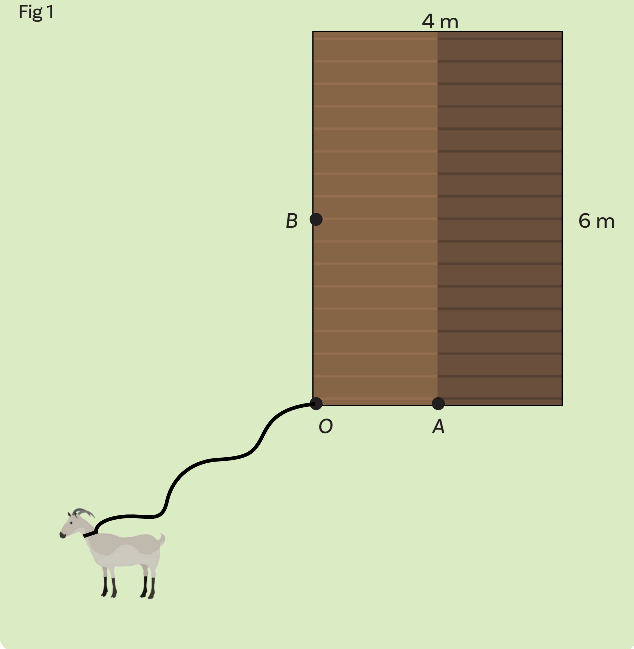
The diagram in fig 1 shows how this arrangement looks from above – make sure the students appreciate that this is a plan view. The height of the shed is irrelevant, so long as the goat can't climb over it!

Q *Suppose that the rope is 3 metres long. Make a sketch that clearly highlights the grass that the goat can reach. Then calculate the area of the grass that the goat can eat.*

Students need to see that the goat can reach three quarters of a circle with a radius of 3m centred on O, and can therefore eat $\frac{3}{4}\pi 3^2 =$ 21.2 m^2 (correct to 1 decimal place) of grass.

Q *What happens for other lengths of rope? See Fig 1 – what happens if the goat is tethered at A or B, rather than O? From which tethering point can it eat the most grass?*

Fig 1



DISCUSSION

Q *What did you find out? What happens as the rope gets longer? What happens if you change the tethering point from O to A or B? How did you work it out?*

Students might begin by making rough sketches of the locus for different lengths of rope. Alternatively, they could produce accurate scale-drawings – with, say, 1 cm representing 1 m – perhaps using centimetre-squared paper, to make drawing the shed easier. They will need to think carefully about what happens when a rope snags on a corner of the shed.

For the goat tethered at O, and a rope of length r metres (r for *r*ope or *r*adius), there are three initial cases, as shown in Fig 2.

This makes a *continuous, piecewise function*. Confident students might want to check, via substituting in, that 'adjacent' formulae give the same value where they 'meet' (at $r = 4$ and $r = 6$). They could also attempt to sketch a graph as shown in Fig 3, in which the three cases are represented by different colours. Similar thinking will give similar equations and curves for tethering the goat at A or at B. We can see from the graph that for any given length of rope, the goat tethered at O gets the most grass.

Fig 2

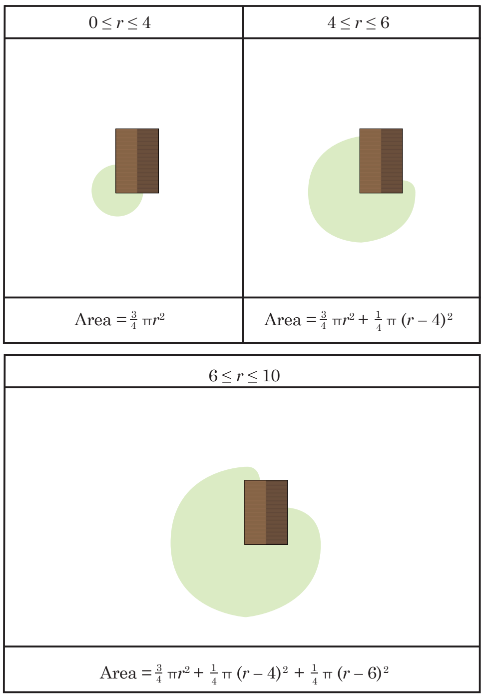
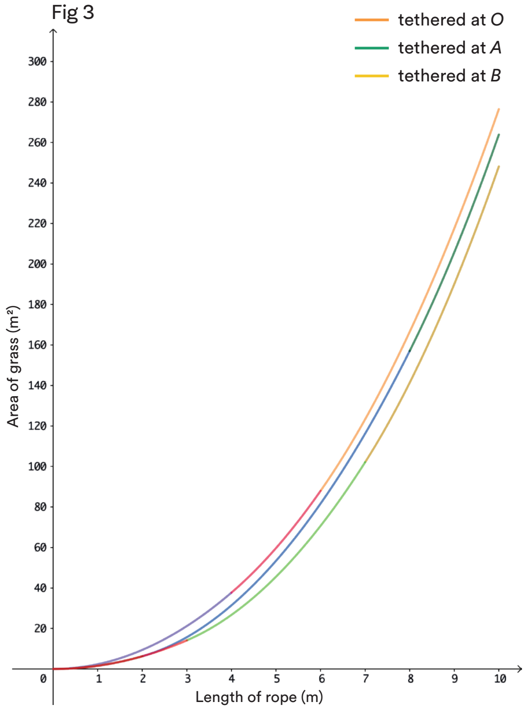


Fig 3



ADDITIONAL RESOURCE

There is a classic, difficult goat-tethering problem that is well-known in recreational mathematics – further details can be found via bit.ly/goat-problem

GOING DEEPER

Confident students could explore what happens with ropes longer than 10 metres or sheds that have different dimensions. They could also invent their own tethered goat problems, perhaps with more complicated shed shapes, or even obstacles in the field!

ABOUT OUR EXPERT

Colin Foster is a Reader in Mathematics Education at the Mathematics Education Centre at Loughborough University. He has written many books and articles for mathematics teachers. His website is www.foster77.co.uk, and on Twitter he is @colinfoster77.