Students need to understand the reasons behind the formulae for the area of different quadrilaterals, says Colin Foster.

In this lesson, students will explore ways of calculating the areas of different quadrilaterals, beginning with parallelograms as the key shape, leading to triangles (as half a parallelogram) and squares and rectangles (as special parallelograms). Finally, they will look at trapezia, which, like parallelograms, can also be thought of as formed from two triangles, but in this case the triangles are not congruent. The aim of the lesson is to give students a sound feeling for area, as opposed to simply memorising formulae.

**STARTER ACTIVITY**

*Q* Can you describe these two tables?
These drawings are available at teachwire.net/ks3trapezia to display on the board or hand out on paper.

This is known as the Shepard’s table illusion. Students will probably think that the table on the left is short and fat, whereas the table on the right is long and thin. However, on the page, the two parallelograms are exactly the same shape and size. This is a trick of perspective – students might find this hard to believe and may need you to verify it by dragging one on top of the other!

**TRAPEZIA ACTS**

Why Teach This?
Understanding area is critical to doing geometry.

**Key Curriculum Links**
- derive and apply formulae to calculate and solve problems involving area of trapezia.

How can we find the area of different quadrilaterals?
DISCUSSION

Q What different trapezia did you think of? Did anyone think of a parallelogram or a rectangle or a square? Did anyone use an isosceles trapezium? Or a right-angled trapezium? How do you know that the areas of your trapezia are all definitely 24 cm²?

The intention is that the insights from this lesson mean that students will be thinking more about why the areas are what they are, in terms of shearing and composition from triangles, rather than mindlessly substituting numbers into formulae.

Q We can’t always trust our eyes to tell us how big a shape is. But we can reason whether two shapes are the same area or not. Are these two shapes the same area or different areas? Why?

Students may reason that the areas are the same because the right-hand shape is a 90° rotation of the left-hand shape, and rotation preserves area. Or they might imagine shearing the parallelograms into 5 x 3 rectangles, since shearing preserves area. If students doubt this, a drawing like this might convince them:

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So, if the area of a parallelogram (including rectangles and squares) is $bh$ then the area of a triangle must be $\frac{1}{2}bh$.

This leaves trapezia, which, like parallelograms, are also made up of two triangles, but in this case the two triangles are not necessarily congruent. (If they are, the trapezium is a parallelogram.)

If you think of a general (non-isosceles trapezium) in this way, then it is clear that the area is the sum of the areas of the two triangles:

Once parallelograms are understood, rectangles and squares are sorted (because they are both special cases of rectangles) and triangles are also easy, because every triangle is half of a parallelogram:

I find that this approach is much easier for students than dissection proofs where pieces have to be cut off and rearranged to make rectangles or parallelograms.

Q On squared paper, draw 10 different trapezia that all have an area of exactly 24 cm².

This is a good challenge, especially if students see parallelograms, rectangles and squares (but not triangles!) all as special cases of trapezia, and therefore legitimate responses to this task.

If students see how a parallelogram can always be sheared into a rectangle, then it should be clear that its area is the same as the area of the rectangle it gets sheared into. This helps with seeing the base and the height (not the side length) of the original parallelogram that need to be multiplied:

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