Lesson plan: Mathematics KS3

TRIGONOMETRY WITHOUT TEARS

Using a unit circle can help develop a deeper understanding of working with right-angled triangles

In this lesson, students explore the behaviour of a radius vector as it rotates around a unit circle. The coordinates of the end of the radius are (cos θ, sin θ), where θ is the anticlockwise angle from the positive x-axis. This provides a way to introduce students to the sine and cosine ratios as functions of an angle, and avoids the need to memorise SOHCAHTOA and procedures that may be poorly understood.

Q What happens to the coordinates of the endpoint of a line segment as it rotates about the origin?

STARTER ACTIVITY

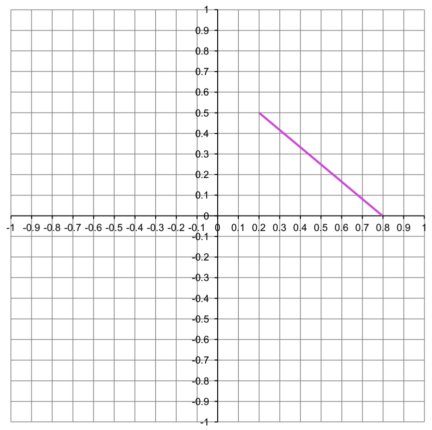
Show students the image, right.
(A downloadable version is available at teachwire.net/ks3trig)

Q *Look at this pink line segment. Can you tell me the coordinates of the ends of the line segment?*

This task is really to check that students can read off decimal coordinates correctly. They may be confused by the order ("along the corridor and up the stairs"; alternatively, "x is a-cross"), as well as by the fact that the numbers are decimals.

Q *The pink line segment is one side of a square which fits completely on the grid. Can you write down the coordinates for the other two vertices?*

This time, some visualisation is needed, and negative numbers are involved. Students may think that the other vertices are (-0.2, -0.5) and (-0.8, 0), but that is not correct.



Read Russell Timmins' easily digestible explanation of trigonometric ratios at **teachwire.net/trigratios**

WHY TEACH THIS?

Trigonometry in right-angled triangles gives students an appreciation of the power of mathematics to precisely calculate lengths that would be tedious to draw and measure

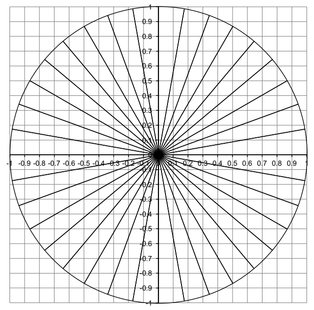
KEY CURRICULUM LINKS

+ Use mathematical language and properties precisely
+ Use trigonometric ratios in similar triangles to solve problems involving right-angled triangles

Colin Foster was recently interviewed on the Mr Barton Maths podcast - listen now at **tinyurl.com/tscfpodcast**

MAIN ACTIVITY

Give students a copy of the task sheet shown below (this is available at **teachwire.net/ks3trig**).



Q Now it's the same grid, but we are looking at a circle. Imagine a line segment starting at the origin (0, 0) and ending at (1, 0) on the positive x-axis. Now imagine it rotating anticlockwise about the origin. The sheet shows you where it ends up after every 10 degrees of rotation.

Make sure that students understand how the diagram was generated, as otherwise all the lines can seem quite overwhelming.

Q I want you to write down the *x* and *y* coordinates of the end of the line segment at each position around the circle.

Students should make a table like this:

Angle	x-coordinate	y-coordinate	y-coordinate / x-coordinate
0°			
10°			
20°			
30°			

…etc

Ask students to fill in their tables, estimating all the values to 2 decimal places, and using a calculator to do the division that's necessary to complete the fourth column.

Encourage students to look carefully at the patterns in their numbers as they go. (Note that for 90° and 270° the value for the fourth column is undefined, since the gradient is ±∞.)

When students have finished, show them how to make sure that their calculators are in 'degrees mode' and then ask them to calculate the **cosine** of each angle, the **sine** of each angle and **tangent** of each angle. The values they obtain should closely match those in the three columns of their tables.

Q What do you think will happen if you plot a graph of each column against the angle? Plot the angle on the horizontal axis.

Encourage students to predict how their graphs will look before plotting them, based on the numbers in their tables, but also on how the radius vector moves around the circle. It may be best to plot the first two graphs on the same axes, but the third graph will require a different vertical scale, and is more complicated, because of the asymptotes at 90° and 270°.

DISCUSSION

You could conclude the lesson by discussing what students have found out:

- cos θ can be defined as the **x-coordinate** of a unit line segment from (0, 0) rotated through an anticlockwise angle of θ from the positive x-axis.
- Similarly, sin θ can be defined as the **y-coordinate** of the same line segment.
- And tan θ can be defined as the **gradient** of the same line segment.

(One way to remember which one is which is that in the alphabet *x* comes before *y* and cos comes before sin.)

You can draw students' attention to the fact that the *x*-distance, the *y*-distance, and the unit line segment itself together always make a right-angled triangle. So the sin, cos and tan buttons on the calculator enable us to find the lengths of sides of a triangle if we know the angle, without having to bother drawing it accurately.

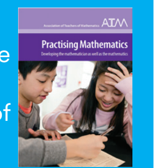
For right-angled triangles with a hypotenuse other than 1, we can just scale the triangle (i.e., the whole circle) by whatever factor is necessary. All of this can be a practical way to solve right-angled triangles, without having to memorise and use SOHCAHTOA. It also prepares the way for future years when students need to see sin, cos and tan as functions of angles, including angles greater than 90°. But for now they could focus on just the first quadrant, where everything is positive.

GOING DEEPER

Confident students could list the similarities and differences in the graphs of cos, sin and tan, and try to explain them in relation to the unit circle.

ADDITIONAL RESOURCES

There are nice animations of this at **https://commons.wikimedia.org/wiki/File:Circle_cos_sin.gif** and **geogebra.org/m/cNEtsbvC**



This new book from the Association of Teachers of Mathematics is full of ideas, tasks and activities for practising the content of the secondary mathematics curriculum. It aims to develop pupils' reasoning and problem solving skills that are so vital for success. The book covers numerous areas of the mathematics curriculum, and is accompanied by a set of resource cards. **www.atm.org.uk/shop/act107pk**

THE AUTHOR

Colin Foster is an Associate Professor in mathematics education in the School of Education at the University of Leicester. He has written many books and articles for mathematics teachers (see www.foster77.co.uk).