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Teaching Tip: How to Manipulate Test Scores

Colin Foster (c@foster77.co.uk)

Suppose we administer an examination with two sections (A and B) to three students (Ava, Beth and Carl) who obtain the raw scores in Table 1. The mean scores give the ordering: Beth $>$ Carl $>$ Ava.

Table 1.

	A	B	Mean
Ava	2	8	5
Beth	4	7	5.5
Carl	4.5	6	5.25

It turns out that, Ava, our favorite student, has come out worst. All is not lost, however, since we can do some *post hoc* 'analysis'. We cannot change any individual score, but the *weighting* of the two sections is not yet decided. In theory there are $3! = 6$ possible orderings of the three students. Can we, by judicious *fixing* of the weights, obtain whichever of those six orderings we want?

To be still more mathematically minded, can we obtain a preferred ranking regardless of the particular scores of the students? Sadly, the answer to this broader question is *no*. For example, if the students all score zero on both parts, then there is nothing to play with. But under what conditions *is* it possible to arrange that the students' weighted scores come out any way we please?

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Let x be the weight for section A. Section B then gets the complementary weight $1 - x$. If a student scores a on section A and b on section B, their weighted score is $y = ax + b(1 - x) = (a - b)x + b$. For the data in Table 1, this leads to three equations, one for each student:

$$y_A = -6x + 8, \quad y_B = -3x + 7, \quad y_C = -1.5x + 6.$$

The graphs of these equations are in Figure 1. The ordering Beth > Carl > Ava corresponds to the order in which the graphs intersect the dashed line at $x = 0.5$. From the graph it is clear that three other orderings are possible by choosing an appropriate x. This corresponds to translating the dashed line.

Thus we have four possible orders for our three students, but not Ava > Carl > Beth or Carl > Ava > Beth. Since three lines can intersect in at most three points, the greatest number of different orderings we can obtain is four, so our situation is best possible. However, in general, it is *never* possible to obtain all six possible orderings of our three students. The reader may find it amusing to work out the several different cases and which orderings are possible. It depends on the relationship among the three lines, specifically, how many intersections lie in the range $0 \le x \le 1$ and what dependencies there are among the three lines.

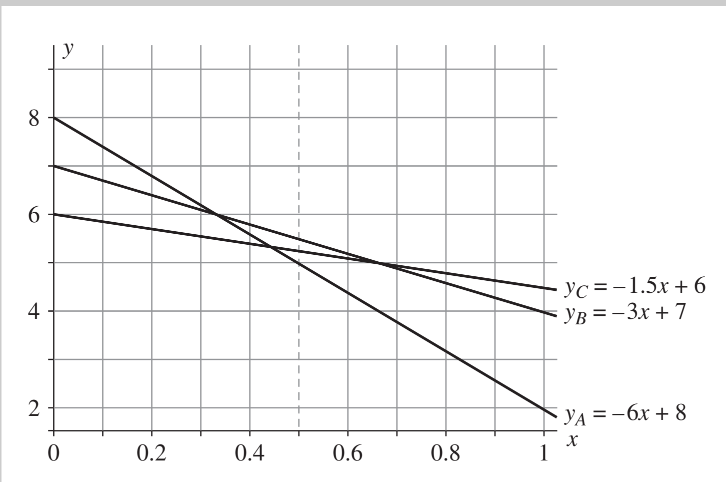

Figure 1.

The good news, from the students' standpoint, is that with more students a teacher's options are reduced dramatically. The maximum number of intersections of n lines is $n(n-1)/2$. Even in the best possible case, in which all these intersections occur for $0 \le x \le 1$, it still provides only $n(n-1)/2 + 1$ rankings out of a possible $n!$. The proportion of available orderings tends to zero as n tends to infinity.

The moral? Students' best protection from teacher manipulation of test results is a large class size!