

FORMATIVE ASSESSMENT LESSONS FOR CONCEPT DEVELOPMENT AND PROBLEM SOLVING

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Formative assessment is the process by which teachers and students gather evidence of learning and then use this to adapt the way they teach and learn in the classroom. In this paper we describe a design research project in which we integrated formative assessment into mathematics classroom materials. We outline two examples of formative assessment lessons, one concept-based and the other problem-solving, highlighting the important roles within them of pre-assessment, formative feedback questions and sample work for students to critique.

INTRODUCTION

High quality *formative* classroom assessment has the potential to produce substantial student learning gains (Black, Harrison, Lee, Marshall, & Wiliam, 2003; Black & Wiliam, 1998, 1999). We follow Black and Wiliam's (2009) definition that:

Practice in a classroom is *formative* to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited. (Black & Wiliam, 2009, p. 9)

In 2009, with funding from the Bill & Melinda Gates Foundation, we began the Mathematics Assessment Project (MAP), to support US Middle and High Schools in implementing the new Common Core State Standards for Mathematics (CCSSM). These standards place a renewed focus on conceptual understanding and on the development of practices (or processes) that should permeate all mathematical activity. In this project, we explored the research question: How can well-designed materials enable teachers to make high-quality formative assessment an integral part of the implemented curriculum in their classrooms, even where linked professional development support is limited or non-existent?

The project has since developed over 100 formative assessment lessons, called *Classroom Challenges*. Each lesson consists of student resources and an extensive teacher guide¹. In this paper we describe the research-based design of these materials and outline two examples, one concept-based and the other focused on supporting students' problem-solving strategies. We highlight across both kinds of lesson the important roles of pre-assessment, formative feedback questions and pre-designed sample work for students to critique. We also articulate how the interaction between the three agents of formative assessment (teachers, learners and their peers) and the three main aspects of formative assessment (identifying where learners are in their learning, where they are going, and how to bridge the gap) is manifest in the lesson structures. This interaction, or the "five key strategies" of formative assessment (Table 1), was first articulated by Wiliam and Thompson

¹ These are available on the website: <http://map.mathshell.org.uk/materials/index.php>.

(2007). Finally, we offer evidence from early independent studies that suggest that these lessons can indeed enable teachers to redefine the classroom contract.

	<i>Where the learner is going</i>	<i>Where the learner is right now</i>	<i>How to get there</i>
Teacher	1. Clarifying learning intentions and criteria for success	2. Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding	3. Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	4. Activating students as instructional resources for one another	
Learner	Understanding and sharing learning intentions and criteria for success	5. Activating students as the owners of their own learning	

Table 1: Key strategies of formative assessment

THEORETICAL FRAMEWORK FOR TASK DESIGN

The CCSSM make it clear that the goals of the new curriculum are to foster a deeper, connected conceptual understanding of mathematics, along with the strategic skills necessary to tackle non-routine problems. In our work, we found it necessary to distinguish between lessons that are designed to foster conceptual development and those that are designed to develop problem-solving strategies. In the former, the focus of student activity is on the analysis and discussion of different *interpretations* of mathematical ideas, while in the latter the focus is on discussing and comparing alternative *approaches* to problems. The intention was that concept lessons might be used partway through the teaching of a particular topic, providing the teacher with opportunities to assess students’ understanding and time to respond adaptively. Problem-solving lessons were designed to be used more flexibly, for example between topics, to assess how well students could select already familiar mathematical techniques to tackle unfamiliar, non-routine problems, and thus provide a means for improving their strategic awareness.

The tasks we selected for the concept-based *Classroom Challenges* were designed to foster collaborative sense-making. Sierpiska (1994) suggests that people feel that they have understood something when they have achieved a sense of order and harmony, where there is a sense of a ‘unifying thought’, of simplification, of seeing an underlying structure and a feeling that the essence of an idea has been captured. She lists four mental operations involved in understanding: “identification: we can bring the concept to the foreground of attention, name and describe it; discrimination: we can see similarities and differences between this concept and others; generalisation: we can see general properties of the concept in particular cases of it; synthesis: we can perceive a unifying principle” (p. 32). To these we add the notion of representation: when we understand something, we are able to characterise it in a variety of ways: verbally, visually and/or symbolically. In the light of this framework, we developed four genres of tasks for our concept-development lessons (Table 2). The first two rows refer to activities with mathematical objects (classifying and representing them), the third refers to making conjectures and statements about those objects, while the fourth refers to the identification of situations within which those objects may be found.

<i>Assessment task genres</i>	<i>Sample classroom activities</i>
<i>Classify and define mathematical objects and structures</i>	Identifying and describing attributes and sorting objects accordingly. Creating and identifying examples and non-examples. Creating and testing definitions.
<i>Represent and translate between mathematical concepts and their representations</i>	Interpreting a range of representations including diagrams, graphs and formulae. Translating between representations and studying the co-variation between representations.
<i>Justify and/or prove mathematical conjectures, procedures and connections</i>	Making and testing mathematical conjectures and procedures. Identifying examples that support or refute a conjecture. Creating arguments that explain why conjectures and procedures may or may not be valid.
<i>Identify and analyse structure within situations</i>	Studying and modifying mathematical situations. Exploring relationships between variables. Comparing and relating mathematical structures.

Table 2: Assessment task genres for concept development

The problem-solving *Classroom Challenges* were designed to assess and improve the capability of students to solve multi-step, non-routine problems and to extend this to the formulation and tackling of problems from the real world. We define a problem as a task that the individual wants to tackle, but for which he or she does not have access to a straightforward means of solution (Schoenfeld, 1985). One consequence of this definition is that it is pedagogically inconsistent to design problem-solving tasks for the purpose of practising a procedure or developing an understanding of a particular concept. In order to develop strategic competence, students must be free to experiment with a range of approaches. They may or may not decide to use any particular procedure or concept; these cannot be pre-determined. Problem solving is contained within the broader processes of mathematical modelling. Modelling additionally requires the formulation of problems by, for example, restricting the number of variables and making simplifying assumptions. Later in the process, solutions must be interpreted and validated in terms of the original context. Some task genres and sample classroom activities for strategic competence are shown in Table 3.

<i>Assessment task genres</i>	<i>Sample classroom activities</i>
<i>Solve a non-routine problem by creating an extended chain of reasoning</i>	Selecting appropriate mathematical concepts and procedures. Planning an approach. Carrying out the plan, monitoring progress and changing direction, where necessary. Reflecting on solutions; examining for reasonableness within the context. Reflecting on strategy; where might it have been improved?
<i>Formulate and interpret a mathematical model of a situation that may be adapted and used in a range of situations</i>	Making suitable assumptions to simplify a situation. Representing a situation mathematically. Identifying significant variables in situations. Generating relationships between variables. Identifying accessible questions that may be tackled within a situation. Interpreting and validating a model in terms of the context.

Table 3: Assessment task genres for problem-solving lessons

DESIGN-BASED METHODOLOGY

Our method for lesson design was based on design-research principles, involving theory-driven iterative cycles of design, enactment, analysis and redesign (Barab & Squire, 2004; Bereiter, 2002; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; DBRC, 2003; Kelly, 2003; van den Akker, Gravemeijer, McKenney, & Nieveen, 2006). In contrast to much design research, we worked to ensure that the products were robust in large-scale use by fairly typical end-users – thus engaging in what Burkhardt (2006) has termed “engineering research”.

Each lesson was developed, through two iterative design cycles, and trialled in three or four US classrooms between each revision. This sample size enabled us to obtain rich, detailed feedback, while also allowing us to distinguish general implementation issues from more idiosyncratic variations by individual teachers. Revisions were based on structured, detailed feedback from experienced local observers in California, Rhode Island and the Midwest. Overall we obtained approximately 700 observer reports of lessons, from over 100 teachers (over 50 schools) using these materials. We also observed many of the lessons first-hand in UK schools. On this basis the lessons were revised. These lessons have subsequently been researched by other independent organisations (see Conclusion section).

We now describe two examples of the formative assessment lessons that we developed, one concept-based and the other problem-solving. Each lesson was designed to occupy about 45 minutes of classroom time and to be worked on collaboratively. Complete lesson guides for these lessons may be freely downloaded from <http://map.mathshell.org>.

A CONCEPT-DEVELOPMENT LESSON

The objective of this lesson, *Distance-Time Graphs*, is to provide a means for a teacher to formatively assess students’ capacity to interpret graphs. The lesson is preceded by a short diagnostic assessment, designed to expose students’ prior understandings and interpretations (Fig. 1). We encourage teachers to prepare for the lesson by reading through students’ responses and by preparing probing questions that will advance student thinking. They are advised not to score or grade the work. Through our trials of the task, we developed a “common issues table” (Fig. 1) that forewarns teachers of some common interpretations students may have, and suggests questions that the teacher might pose to advance thinking. This form of feedback has been shown to be more powerful than grades or scores, which detract from the mathematics and encourage competition rather than collaboration (Black et al., 2003; Black & Wiliam, 1998). Some teachers write their questions on the student work while others prepare short lists of questions for the whole class to consider.

The lesson itself is then structured in five parts:

- 1. Make existing concepts and methods explicit.** The lesson begins with an initial task to clarify the learning intentions, create curiosity, help students become aware of their own intuitive interpretations and model the level of reasoning expected in the main activity (strategy 1)². The teacher invites and probes explanations but does not correct students or attempt to reach resolution at this point.

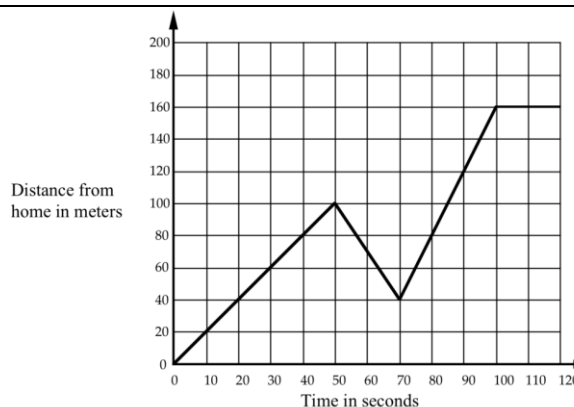
² The strategy numbers refer to the formative assessment strategies listed in Table 1.

2. **Collaborative activity: Matching graphs, stories and tables.** This phase is designed to create student-student discussions in which they share and challenge each other's interpretations (strategy 2). Ten distance/time graphs are to be matched with nine 'stories' (the tenth to be constructed by the student). When the cards have been discussed and matched, the teacher distributes a further set of cards that contain distance/time tables of numerical data. These provide feedback by enabling students to check their own responses and reconsider the decisions that have been made. Students collaborate to construct posters displaying their reasoning. While students work, the teacher is encouraged to ask the prepared questions from the initial diagnostic assessment (strategy 3).
3. **Inter-group discussion: Comparing interpretations.** Students' posters are displayed, and students visit each other's posters and check them, demanding explanations (strategy 4).
4. **Plenary discussion.** Students revisit the task that was introduced at the beginning of the lesson and resolution is now sought. Drawing on examples of student work produced during the lesson, the teacher directs attention to the significant concepts that have arisen (strategy 2).
5. **Individual work: Improving solutions to the pre-assessment task.** Students now revisit the work they did on the pre-assessment task. They describe how they would now answer the task differently and write about what they have learned (strategy 5).

Journey to the bus stop

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 metres. The graph shows his journey on one particular day.

1. Describe what may have happened. Include details like how fast he walked.
2. Are all sections of the graph realistic? Fully explain your answer.



Issue	Suggested questions and prompts
Student interprets the graph as a picture For example: The student assumes that as the graph goes up and down, Tom's path is going up and down <i>or</i> assumes that a straight line on a graph means that the motion is along a straight path.	If a person walked in a circle around their home, what would the graph look like? If a person walked at a steady speed up and down a hill, directly away from home, what would the graph look like? In each section of his journey, is Tom's speed steady or is it changing? How do you know? How can you figure out Tom's speed in each section of the journey?
Student interprets the graph as speed-time The student interprets a positive slope as 'speeding up' and a negative slope as 'slowing down'.	If a person walked for a mile at a steady speed, away from home, then turned round and walked back home at the same steady speed, what would the graph look like? How does the distance change during the second section of Tom's journey? What does this mean? How can you tell if Tom is traveling away from or towards home?

Figure 1: Initial assessment task: *Journey to the bus stop*, and an extract from the associated 'Common issues table'

A PROBLEM-SOLVING LESSON

Our trials showed that teachers find it difficult to interpret, monitor and select students' extended reasoning during a problem-solving lesson. We therefore decided to precede each lesson with a preliminary assessment in which students tackle a substantial problem individually. The teacher reviews a sample of the students' initial attempts and identifies the main issues that need addressing. The focus is on *approaches* to the problem. If time permits, teachers write feedback questions on each student's work, or alternatively prepare questions for the whole class to consider.

For example, one High School problem poses the following question: "A poster asserts that one female cat can have 2,000 descendants in 18 months. Is this realistic?" This problem is accompanied by five pieces of information: "The length of a cat's pregnancy is about 2 months"; "Cats can first get pregnant when they are 4 months old"; "Cats normally have 4 to 6 kittens in a litter"; "A female cat can have about 3 litters per year"; "Cats stop having kittens when they are 10 years old."³

The lesson is structured as follows:

- 1. Introduction: *Responding to formative feedback.*** The teacher re-introduces the main task for the lesson and returns students' initial attempts, along with some formative questions. Students have a few minutes to read these questions and respond to them individually (strategy 3). 'Common issues' have been identified from trials and these are provided for teachers to use (Fig. 2).

<i>Issue</i>	<i>Suggested questions and prompts</i>
Has difficulty starting	Can you describe what happens during the first five months?
Does not develop a suitable representation	Can you make a diagram or table to show what is happening?
Work is unsystematic	Could you start by just looking at the litters from the first cat? What would you do after that?
Develops a partial model	Do you think the first litter of kittens will have time to grow and have litters of their own? What about their kittens?
Does not make reasonable assumptions	What assumptions have you made? Are all your kittens born at the beginning of the year? Are all your kittens females?

Figure 2: An extract from the 'Common issues table' for *Having Kittens*

- 2. Group work: *Comparing strategic approaches.*** Students are asked to discuss the work of each individual in small groups and then produce a poster showing a joint solution that is better than the individual attempts. Groups are organised so that students with contrasting ideas are paired, thus promoting peer assessment (strategy 4). The teacher's role is to observe groups and challenge students using the prepared questions to refine and improve their strategies (strategy 2). The teacher may at this point ask students to review the strategic approaches produced by other groups in the class, and justify their own. Additionally, the teacher may introduce up to four pieces of "pre-designed sample student work" (Evans & Swan, 2014), provided in the materials, which are chosen to highlight alternative approaches. Each piece of work is annotated with questions that focus students' attention (Fig. 3).

³ This task was originally designed by Lesley Ravenscroft and appears courtesy of the Bowland Charitable Trust.

3. **Group work: Refining solutions.** Students revisit the task and try to use insights to further refine their solution (strategy 4).
4. **Whole-class discussion: Reviewing learning.** The teacher holds a plenary discussion to focus on the processes involved in the problem, such as the implications of making different assumptions, the power of alternative representations and the general mathematical structure of the problem.

<p>6 months</p> <p>6 months</p> <p>6 months</p> <p>Mum</p> <p>6</p> <p>6</p> <p>6</p> <p>6</p> <p>6</p> <p>6</p> <p>36</p> <p>36</p> <p>36</p> <p>36</p> <p>36</p> <p>36</p> <p>Total cats = $1 + 6 \times 6 + 6 \times 36$ $= 1 + 36 + 216$ $= 253$ So its not realistic</p>	<p>Questions for students</p> <ul style="list-style-type: none"> • What has Wayne done correctly? • What assumptions has he made? • How can Wayne's work be improved? <p>Notes from the teacher guide</p> <p>Wayne has assumed that the mother has six kittens after 6 months, and has considered succeeding generations. He has, however, forgotten that each cat may have more than one litter. He has shown the timeline clearly. Wayne doesn't explain where the 6-month gaps have come from.</p>
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Figure 3: Sample work for discussion, with commentary from the teacher guide

CONCLUSION

The two lessons described above contains many features that are not common in mathematics teaching, at least in the US and UK. In both kinds of lesson, there is a strong emphasis on the use of preliminary formative assessment, which enables the teacher to prepare for and adapt interventions to the student reasoning that will be encountered. Students spend much of the lesson in dialogic talk, focused on comparing mathematical processes. The successive opportunities for refining the solution enable students to pursue multiple methods and to compare and evaluate them. Finally, pre-designed sample student work is used to foster the development of critical competence.

Early evidence of the impact of these lessons is encouraging. Drawing on a national survey of 1239 mathematics teachers from 21 states, and interview data from four sites, Research for Action (RFA)⁴, found that a large majority of teachers reported that the use of the *Classroom Challenges* had helped them to implement the Common Core State Standards, raise their expectations for students, learn new strategies for teaching subject matter, use formative assessment and differentiate instruction.

The National Center for Research on Evaluation, Standards and Student Testing (CRESST) examined the implementation and impact of *Classroom Challenges* in 9th-grade Algebra 1 classes (Herman et al., 2015). This study used a quasi-experimental design to compare student performance with *Classroom Challenges* to a matched sample of students from across Kentucky comparable in prior achievement and demographic characteristics. On average, study teachers implemented only four to six *Challenges* during the study year (or 8-12 days), yet, relative to typical growth in mathematics from eighth to ninth grade, the effect size for the *Classroom Challenges* represented an additional 4.6 months of schooling. Although teachers felt that that the *Challenges* benefited

⁴ RFA is a non-profit research organisation; see: <http://www.researchforaction.org/rfa-study-of-tools-aligned-ccss/>

students' conceptual understanding and mathematical thinking, they reported that sizeable proportions of their students struggled, and it appeared that lower-achieving students benefited less than higher achievers. This, they suggested, may be due to the great difference in challenge and learning style required by these lessons, compared with students' previous diet of largely procedural learning.

Finally, Inverness Research (IR, 2015) in 2014 surveyed 636 students from 31 trial classes (6th grade to High School) across five states in the US. They found that the majority of students enjoyed learning mathematics through these lessons and reported that they understood it better, had increased in their participation, and had improved in listening to others and in explaining their mathematical thinking. About 20%, however, remained unaffected by or disaffected with these lessons. This was because they did not enjoy working in groups, they objected to the investigative approach and/or they felt that these lessons were too long or too difficult.

In conclusion, *Classroom Challenges* appear to provide a model for teachers as they attempt to introduce formative assessment into their everyday classroom practice, but they require a radical shift in the predominant classroom culture. How far teachers transfer this approach into the rest of their teaching is the focus of ongoing research.

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