1.9 Percentages

- There's nothing fundamental about dividing by 100 – we choose to invent % because it's convenient. Pupils need to see this convenience early on (e.g., task 1.9.1). For very small proportions it's more convenient to use ppm (parts per million) instead.
- % increase and decrease seem to work best using "decimal multipliers"; i.e., to increase by 12% multiply by 1.12 and to decrease by 12% multiply (not divide) by 0.88.
- To work out the % increase or decrease given the starting amount and finishing amount (or either and the difference), always work out $\frac{\text{new amount}}{\text{old amount}}$ (this is the "multiplier"). If this comes to, say, 1.32, it's been a 32% increase. To find an "old amount" from the "new amount" (the trickiest situation) you need to *divide by the multiplier*. (The sheet "Percentage Increase and Decrease" assumes this approach.)

1.9.1 Write on the board the marks I might have got at school in my report:

Eng $\frac{6}{10}$	Maths $\frac{19}{25}$	Sci $\frac{19}{30}$
History $\frac{9}{20}$	Geog $\frac{9}{15}$	RE $\frac{35}{50}$
PE $\frac{2}{3}$	Music $\frac{4}{8}$	French $\frac{3}{5}$

Imagine I go home and my parents say, "You're obviously good at Geography, because you got 9, but not so good at PE because you only got 2".
Why might that not be right?

What's my best/worst subject?
What am I better at – history or geog? maths or science?
Put my subjects in order.

Converting them all into %'s makes it much easier to make comparisons.
Calculators not really needed.

Eng 60%	Maths 76%	Sci 63%
History 45%	Geog 60%	RE 75%
PE 67%	Music 50%	French 60%

Of course, the tests in the different subjects might not be of the same "difficulty". (How can you compare "difficulty" in two different subjects, anyway? We might want to know what the class median was in each, though)

Order (best to worst): Maths-76 (of course!), RE-75, PE-67, Sci-63, Eng=Geog=French-60, Music-50, History-45.

It's helpful to discuss comparisons;
e.g., $\frac{9}{15}$ must be better than $\frac{9}{20}$, etc.

1.9.2 Discuss where you come across percentages apart from in maths lessons; e.g., how are they used in shops?

What do you think about a disinfectant that says "Kills 101% of germs"?
What about a sports coach who says he wants the players to "give 110%"?
Music exams are out of 150 but people sometimes say "percent" if the mark is under 100, yet it isn't really a %.

Can % be more than 100 or is that impossible? When does it make sense to have more than 100% and when does it not make sense?

What's the difference between "increased *by* 200% of its original value" and "increased *to* 200% of its original value"?

10% off price, "100% fat free", "100% cotton", etc., exams, pie charts.

This is just hyperbole – not mathematically accurate.

110% may not be as silly as it sounds – it may mean give 10% more than usual (10% increase).

It is possible when something ends up bigger than it started; e.g., "the crowd this year is 120% of what it was last year" means a 20% increase.
Drawing a number-line from 0 to about 350% and marking decimals 0 to 3.5 is helpful in seeing that %'s are just numbers.

"by" means now 3 × what it was
"to" means now 2 × what it was

1.9.3 If the base of a rectangle is increased by 10% and the area is unchanged, by what percentage is the width *decreased*?

Answer: $\frac{1}{1.1} = 0.909...$, so a decrease of about 0.09 or 9%.

1.9.4 The difference between increasing a number by 7% and decreasing it by 8% is 30. What is the number? Pupils can make up some puzzles like this.

*Answer: 15% of the number = 30, so number = 30 ÷ 0.15 = **200** (check: 214 – 184 = 30)*

1.9.5 The original price of an item is £15. After two mark-ups the final price is £20.70. If the 1ˢᵗ mark-up is 15%, what is the other?
What if it was the 2ⁿᵈ mark-up that was 15%? What would the 1ˢᵗ have been?

Easy with multipliers:
$15.00 \times 1.15 \times m = 20.70$
$m = 1.2$, so the mark-up is 20%.

It doesn't matter – you get the same answer.

1.9.6 Which is bigger 72% of £48 or 48% of £72?

This can be useful because a hard % of an easy number can be written as an easy % of a hard number; e.g., 73% of £50 = 50% of £73 = £36.50.

They're the same, but can pupils explain why? One way to look at it is that
$0.72 \times 48 = 0.48 \times 72$

1.9.7 If I decrease an amount by 10% and then increase the new amount by 10% have I got back the amount I started with?
(Easy to embellish with a context!)

No – the 1ˢᵗ 10% was 10% of a bigger amount than the 2ⁿᵈ 10% was, so what I added on was smaller than what I took away.
If I started with p I now have $1.1 \times 0.9 \times p = 0.99 \times p$ (i.e., I've lost 1% of what I had).

What if I do it in the opposite order?

This time it's $0.9 \times 1.1 \times p = 0.99 \times p$; i.e., the same answer.

1.9.8 If 60% of the school like swimming, 65% like football, 70% like rugby, 75% like hockey and 80% like basketball, what percentage like them all?

Could draw a Venn (1834-1923) diagram.

Answer: Total = 60+65+70+75+80 = 350%. If you share them out as equally as possible everyone has to like at least 3, and at least 50% have to like all four. The maximum % that could like all four would be 60%, so the answer is between 50% and 60%.

1.9.9 Carbon Monoxide (CO) is a poisonous gas which is found in the air and which some people are concerned about. Carbon monoxide detectors measure the concentration of CO in the air in ppm (parts per million).
What is 100 ppm as a percentage?

Answer: 1 ppm is $\frac{1}{1000000}$ = 0.0001%, so 10 000 ppm = 1% and 100 ppm = 0.01%.

A more poisonous gas is hydrogen cyanide (HCN) which is dangerous at a level of 10 ppm = 0.001%.

1.9.10 VAT. A ready reckoner (see sheet) can be useful in a situation where you always want the same percentage of different amounts.

Conveniently, 17.5% = 10% + 5% + 2.5%, and each of these is half the previous amount. This can help with non-calculator calculations.

1.9.11 Work out the percentage of your life you have spent on different things.
e.g., percentage of your life asleep/watching TV/at parties/shopping/doing a particular sport/practising a musical instrument, etc.;
percentage of the school year which is holiday;
percentage of school lesson time given over to maths (if year 7, compare with primary school); percentage of your life spent in assemblies; percentage of your life you've spent as a teenager; percentage of your life spent in a particular country (if relevant).

Some of these aren't too hard, because you can work out per 24 hours or per week.

It's worth thinking through the kinds of assumptions we make when doing these sorts of estimates.

Percentage Increase and Decrease

Fill in the gaps in the table.
The first one is done already.

	old price	new price	new price / old price	what's happened?
1	£34.00	£50.00	1.47	47% increase
2	£6.50	£7.20		
3	£8.50	£8.10		
4	£241.00			41% decrease
5	£78.20			4% increase
6	£1.60		1.24	
7	£852.10		0.30	
8	£29.00			32% decrease
9	£43.80			90% increase
10	£329.35	£400.00		
11	£22.00	£10.00		
12		£179.00	0.90	10% decrease
13		£4.00	1.15	15% increase
14		£11.00		16% increase
15		£11.11		8% decrease
16		£2.00		33% decrease
17		£1,499.00		17% increase
18		£8.50	1.09	
19	£8.00			10% decrease
20		£543.00		17% decrease

Percentage Increase and Decrease ANSWERS

Fill in the gaps in the table.
The first one is done already.

	old price	new price	$\dfrac{\text{new price}}{\text{old price}}$	what's happened?
1	£34.00	£50.00	1.47	47% increase
2	£6.50	£7.20	**1.11**	**11% increase**
3	£8.50	£8.10	**0.95**	**5% decrease**
4	£241.00	**£142.19**	**0.59**	41% decrease
5	£78.20	**£81.33**	**1.04**	4% increase
6	£1.60	**£1.98**	1.24	**24% increase**
7	£852.10	**£255.63**	0.30	**70% decrease**
8	£29.00	**£19.72**	**0.68**	32% decrease
9	£43.80	**£83.22**	**1.90**	90% increase
10	£329.35	£400.00	**1.21**	**21% increase**
11	£22.00	£10.00	**0.45**	**55% decrease**
12	**£198.89**	£179.00	0.90	10% decrease
13	**£3.48**	£4.00	1.15	15% increase
14	**£9.48**	£11.00	**1.16**	16% increase
15	**£12.08**	£11.11	**0.92**	8% decrease
16	**£2.99**	£2.00	**0.67**	33% decrease
17	**£1,281.20**	£1,499.00	**1.17**	17% increase
18	**£7.80**	£8.50	1.09	**9% increase**
19	£8.00	**£7.20**	**0.90**	10% decrease
20	**£654.22**	£543.00	**0.83**	17% decrease

Ready Reckoner for 17.5%

17.5%   17.5%

£	£	£	£	£	£	£	£	£	£	p	p	p	p
1	0.18	51	8.93	101	17.68	152	26.60	310	54.25	1	0	51	9
2	0.35	52	9.10	102	17.85	154	26.95	320	56.00	2	0	52	9
3	0.53	53	9.28	103	18.03	156	27.30	330	57.75	3	1	53	9
4	0.70	54	9.45	104	18.20	158	27.65	340	59.50	4	1	54	9
5	0.88	55	9.63	105	18.38	160	28.00	350	61.25	5	1	55	10
6	1.05	56	9.80	106	18.55	162	28.35	360	63.00	6	1	56	10
7	1.23	57	9.98	107	18.73	164	28.70	365	63.88	7	1	57	10
8	1.40	58	10.15	108	18.90	166	29.05	370	64.75	8	1	58	10
9	1.58	59	10.33	109	19.08	168	29.40	380	66.50	9	2	59	10
10	1.75	60	10.50	110	19.25	170	29.75	390	68.25	10	2	60	11
11	1.93	61	10.68	111	19.43	172	30.10	400	70.00	11	2	61	11
12	2.10	62	10.85	112	19.60	174	30.45	410	71.75	12	2	62	11
13	2.28	63	11.03	113	19.78	175	30.63	420	73.50	13	2	63	11
14	2.45	64	11.20	114	19.95	176	30.80	430	75.25	14	2	64	11
15	2.63	65	11.38	115	20.13	178	31.15	440	77.00	15	3	65	11
16	2.80	66	11.55	116	20.30	180	31.50	450	78.75	16	3	66	12
17	2.98	67	11.73	117	20.48	182	31.85	460	80.50	17	3	67	12
18	3.15	68	11.90	118	20.65	184	32.20	470	82.25	18	3	68	12
19	3.33	69	12.08	119	20.83	186	32.55	480	84.00	19	3	69	12
20	3.50	70	12.25	120	21.00	188	32.90	490	85.75	20	4	70	12
21	3.68	71	12.43	121	21.18	190	33.25	500	87.50	21	4	71	12
22	3.85	72	12.60	122	21.35	192	33.60	550	96.25	22	4	72	13
23	4.03	73	12.78	123	21.53	194	33.95	600	105.00	23	4	73	13
24	4.20	74	12.95	124	21.70	196	34.30	650	113.75	24	4	74	13
25	4.38	75	13.13	125	21.88	198	34.65	700	122.50	25	4	75	13
26	4.55	76	13.30	126	22.05	200	35.00	750	131.25	26	5	76	13
27	4.73	77	13.48	127	22.23	204	35.70	800	140.00	27	5	77	13
28	4.90	78	13.65	128	22.40	205	35.88	850	148.75	28	5	78	14
29	5.08	79	13.83	129	22.58	210	36.75	900	157.50	29	5	79	14
30	5.25	80	14.00	130	22.75	215	37.63	950	166.25	30	5	80	14
31	5.43	81	14.18	131	22.93	216	37.80	1000	175.00	31	5	81	14
32	5.60	82	14.35	132	23.10	220	38.50	1016	177.80	32	6	82	14
33	5.78	83	14.53	133	23.28	225	39.38	1250	218.75	33	6	83	15
34	5.95	84	14.70	134	23.45	228	39.90	1500	262.50	34	6	84	15
35	6.13	85	14.88	135	23.63	230	40.25	1728	302.40	35	6	85	15
36	6.30	86	15.05	136	23.80	235	41.13	1750	306.25	36	6	86	15
37	6.48	87	15.23	137	23.98	240	42.00	1760	308.00	37	6	87	15
38	6.65	88	15.40	138	24.15	245	42.88	2000	350.00	38	7	88	15
39	6.83	89	15.58	139	24.33	250	43.75	2240	392.00	39	7	89	16
40	7.00	90	15.75	140	24.50	255	44.63	2500	437.50	40	7	90	16
41	7.18	91	15.93	141	24.68	260	45.50	3000	525.00	41	7	91	16
42	7.35	92	16.10	142	24.85	265	46.38	4000	700.00	42	7	92	16
43	7.53	93	16.28	143	25.03	270	47.25	5000	875.00	43	8	93	16
44	7.70	94	16.45	144	25.20	275	48.13	6000	1050.00	44	8	94	16
45	7.88	95	16.63	145	25.38	280	49.00	7000	1225.00	45	8	95	17
46	8.05	96	16.80	146	25.55	285	49.88	8000	1400.00	46	8	96	17
47	8.23	97	16.98	147	25.73	288	50.40	9000	1575.00	47	8	97	17
48	8.40	98	17.15	148	25.90	290	50.75	10000	1750.00	48	8	98	17
49	8.58	99	17.33	149	26.08	295	51.63	15000	2625.00	49	9	99	17
50	8.75	100	17.50	150	26.25	300	52.50	20000	3500.00	50	9	100	18