1.12 Priority of Operations

- **BIDMAS** may make more sense than BODMAS.

Brackets
Indices
Division **M**ultiplication
Addition **S**ubtraction

Indices means powers and roots.

Where 2 operations have the same level of priority we work from left to right in the sum.

The line in division behaves like a pair of brackets; e.g., $\frac{10-4}{2}=3$, not 1 or 8.

1.12.1 Write up something like $2+3\times 4$ on the board. What is the answer?

How might different people give different answers to this?

Try it on a calculator to see what happens.

Answers: 20 or 14 depending on which operation you did first.
Introduce the word "operation".
In maths we don't like ambiguities – we want the same answer anywhere in the world, any time, no matter who does the question.
A scientific calculator gives 14; a basic calculator gives 20.

1.12.2 It's important that pupils can get the right answers using the bracket and memory keys on the calculator.
Put complicated combinations on the board. What different answers do we get?
What does BIDMAS make it?

e.g., $\frac{25-(13-4)}{7-3\times 2+1}\times\sqrt{\frac{8\times 3^2}{11-(8+1)}}$

Fraction lines and square root symbols imply brackets around what they enclose.

Pupils can invent these.

Answer: 6 × 8 = 48

1.12.3 Setting out this work as clearly as possible will help – boxing (or colouring/highlighting) the portion you're working out and then writing the whole thing underneath with only the highlighted portion changed (into the answer) and then highlighting the next thing to do (see right).

Example: (keeping one = sign per line)

$$\frac{36-(7-3)^2}{4+2\times 3}$$
$$=\frac{36-4^2}{4+2\times 3}$$
$$=\frac{36-16}{4+2\times 3}$$
$$=\frac{36-16}{4+6}$$
$$=\frac{20}{10}$$
$$=2$$

1.12.4 Puzzle pictures (colour in the answers to produce a picture).

Often popular and available in books.

1.12.5 True or false statements.
If false say what the correct value is and how the mistake could have been made.
e.g., 2 + 8 – 4 × 3 = 18

Answer: false, should be –2; the person has just gone left to right, ignoring BIDMAS.

It can be challenging but worthwhile to try to

Pupils can invent these.

identify how errors (deliberate or not) have been made.

1.12.6 Without using any brackets, fill in $+-\times\div$ (as many of each as you like) to make these true.
9 9 9 9 9 = 89
9 9 9 9 9 = 11
9 9 9 9 9 = 108
1 1 1 1 1 = 3
Make up some for someone else.

Answers:
9 × 9 – 9 ÷ 9 + 9 = 89
9 ÷ 9 + 9 + 9 ÷ 9 = 11
9 + 9 × 9 + 9 + 9 = 108
1 + 1 × 1 + 1 × 1 = 3
and other possible solutions.

Can allow concatenation (sticking together adjacent digits; e.g., 4 8 makes 48) if you like.

Can use pupils' years of birth (e.g., 1 9 9 1).

As a challenge (e.g., for homework) make ten totals from the year of your birth. You're not allowed to alter the order of the digits, but you can use $+-\times\div!\sqrt{\ }$, powers, concatenation and brackets (at your discretion). Must obey BIDMAS!

Can restrict to integer answers or not.

1.12.7 By putting in brackets (as many pairs as you like) what different answers can you make to this? (You mustn't change anything else.)
4 × 5 – 2 + 3 × 6 ÷ 2

4 × 5 – 2 + 3 × 6 ÷ 2 = 27
4 × (5 – 2) + 3 × 6 ÷ 2 = 21
4 × (5 – 2 + 3) × 6 ÷ 2 = 72
4 × (5 – 2 + 3 × 6) ÷ 2 = 42
4 × (5 – 2 + 3 × 6 ÷ 2) = 48
… and lots more!

1.12.8 BIDMAS applies to algebra; e.g., $E = mc^2$ isn't $(mc)^2$ and when we write $3x+4$ we assume multiplication of 3 and x before the addition of the 4.

A common error when calculating the area of a circle is to multiply π by the radius and then square the answer. For this reason, $r^2\pi$ is sometimes more successful than πr^2.

1.12.9 **NEED** "Insert the signs" sheets.

Quite difficult.

1.12.10 Four Fours.
Given a maximum of four number 4's and as many + – × ÷ signs as you want, can you make all the numbers from 1 to 20?

Which numbers can be made in more than one way?

Start by trying to make 1 (= 4 ÷ 4).
Could allow $\sqrt{\ }$ sign (hence 2) or ! (factorial).

1 = 4 ÷ 4	*11 = 44 ÷ 4* *if allowed*
2 = 4 ÷ 4 + 4 ÷ 4 *2 = (4+4) ÷ 4* *2 = $\sqrt{4}$*	*12= 4$\sqrt{4}$ + 4* *12 = 4 + 4 + 4* *12 = 4 × 4 – 4*
3 = 4 – 4 ÷ 4	*13 = 44 ÷ 4 + $\sqrt{4}$*
4	*14 = 4 × 4 – $\sqrt{4}$*
5 = 4 + 4 ÷ 4	*15 = 4 × 4 – 4 ÷ 4*
6 = 4 + (4+4) ÷ 4 *6 = 4 + $\sqrt{4}$*	*16 = 4 × 4* *16 = 4 + 4 + 4 + 4* *16 = 4^4 ÷ (4 × 4)*
7 = 4 + 4 – 4 ÷ 4	*17 = 4 × 4 + 4 ÷ 4*
8 = 4 + 4 *8 = 4$\sqrt{4}$*	*18 = 4 × 4 + $\sqrt{4}$*
9 = 4 + 4 + 4 ÷ 4	*19 = 4 × 4 + $\sqrt{4}$ + 4 ÷ 4,* *but **uses five 4's***
10 = 4 + 4 + 4 – $\sqrt{4}$ *10 = 4 + 4 + $\sqrt{4}$*	*20 = 4 × 4 + 4* *20 = 4 × (4 + 4 ÷ 4)* *20 = 4! – 4*

… and many other possibilities …

You can make 19 from four 4's if you do, e.g.,
4 ÷ 4 + !4 + !4
$!4 = 9$, *where $!n$ is the "subfactorial of n", the number of ways of arranging n items so that each one is in the "wrong" place (e.g., the number of ways of putting 4 letters into 4 envelopes so that no letter is in the correct envelope). But if no-one knows about this notation, this might count as "cheating"!*
(See section 3.4.5.)

Another "cheat" for 19 is $4! - \frac{\sqrt{4}}{.4}$ (note the decimal point in the denominator)!

Insert the Signs

Insert + – × ÷ and ( ) signs in between the numbers to make the sums correct.

e.g., 6 6 3 = 8 insert + and ÷
6 + 6 ÷ 3
= 6 + 2
= 8

You are also allowed to put digits next to each other to make numbers (concatenation) (e.g., 1 2 can become 12), but you can't alter the *order* of the digits.

e.g., 1 2 5 2 = 36 insert × and –
12 × (5 – 2) = 36

Try these.

1 4 1 = 3	5 3 6 7 = 1
4 2 7 = 6	8 7 5 5 = 4
5 6 2 = 9	6 4 2 2 = 9
8 2 8 = 12	3 9 1 7 = 16
6 4 9 = 15	9 3 9 4 = 25
8 4 6 = 18	2 7 4 3 = 36
5 1 6 = 21	4 9 4 2 = 49
9 6 4 = 24	6 4 5 8 = 64
7 4 9 = 27	1 8 5 2 = 81
2 4 7 = 30	6 8 5 2 = 100

1 5 5 4 = 101	8 4 9 7 = 0
2 3 5 6 = 102	6 8 4 6 = 11
6 9 2 5 = 103	5 7 9 3 = 22
2 6 7 8 = 104	1 4 9 3 = 33
8 1 2 9 = 105	3 5 2 8 = 44
9 5 6 7 = 106	1 6 2 8 = 55
9 6 8 7 = 107	2 7 4 3 = 66
6 7 4 7 = 108	3 7 4 7 = 77
8 5 6 9 = 109	4 1 4 8 = 88
5 5 7 3 = 110	8 1 3 5 = 99

Insert the Signs **ANSWERS**

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e.g., 1 2 5 2 = 36 insert × and –
12 × (5 – 2) = 36

Try these.

1 × 4 – 1	=	3
42 ÷ 7	=	6
5 + 6 – 2	=	9
8 ÷ 2 + 8	=	12
6 × 4 – 9	=	15
8 + 4 + 6	=	18
5 + 16	=	21
96 ÷ 4	=	24
(7 – 4) × 9	=	27
2 + 4 × 7	=	30

5 – 3 + 6 – 7	=	1
(8 + 7 + 5) ÷ 5	=	4
6 + 4 – 2 ÷ 2	=	9
3 + 91 ÷ 7	=	16
9 + 3 + 9 + 4	=	25
27 × 4 ÷ 3	=	36
(4 + 94) ÷ 2	=	49
6 × 4 + 5 × 8	=	64
1 + 8 × 5 × 2	=	81
6 × 8 + 52	=	100

1 + 5 × 5 × 4	=	101
(2 + 3 × 5) × 6	=	102
6 × 9 × 2 – 5	=	103
26 + 78	=	104
8 × 12 + 9	=	105
9 × (5 + 6) + 7	=	106
9 + (6 + 8) × 7	=	107
6 × (7 + 4 + 7)	=	108
8 × 5 + 69	=	109
5 + 5 × 7 × 3	=	110

8 ÷ 4 – 9 + 7	=	0
68 ÷ 4 – 6	=	11
(57 + 9) ÷ 3	=	22
1 × 4 × 9 – 3	=	33
352 ÷ 8	=	44
1 + 62 – 8	=	55
2 × (7 + 4) × 3	=	66
3 × 7 × 4 – 7	=	77
4 × (14 + 8)	=	88
8 × 13 – 5	=	99