1.23 Co-ordinates and Straight-Line Graphs

- It's worth getting into the habit of always writing co-ordinates in (x, y) form with comma and brackets. Even in oral work you can ask pupils to say "bracket-number-comma-number-bracket" whenever they describe co-ordinates so as to reinforce that they're always given that way.
- Drawing axes is often difficult: things to emphasise include numbering the *lines,* not the spaces; labelling the x and y axes the right way round (the letter " y " has a "vertical" tail to remind you that it goes with the vertical axis); not missing out the lines next to the axes because the numbers are in the way; using the same scale on both axes (unlike in other graph work) so as not to distort angles, shapes and areas.
- For plotting co-ordinates accurately ("along the corridor and up the stairs" helps with the order; x then y is alphabetical order), it's helpful to get pupils up to the board to do this, so that common mistakes can be discussed and avoided. (If you don't have part of the board covered with squares, you could make an acetate out of the 1 cm × 1 cm squared paper page following and use an OHP.)
- It's important to be able to tell what is a linear equation (that will give a straight-line when you draw it) and what isn't. Only if it can be rearranged into $y = mx + c$ or $x = k$ will the graph be a straight line.

1.23.1 Co-ordinate Shapes/Pictures.
These are readily available in many books; e.g., Christmas designs, animals, etc.

You can link this topic to names of quadrilaterals/polygons and kill two birds with one stone.
("Plot these points and join them up – name the resulting polygon.")
Pupils can make up their own.

Good for wall displays.

You need to make sure that it's clear in what order the points must be plotted and whether pupils are to join up each point to the next with a straight line as they go. It doesn't generally work to plot all the points first and then try to join them up in the right order afterwards.

1.23.2 Equations of straight lines (see sheet of axes and thick line suitable for making into acetates).
You can draw on the acetate with washable projector pens or use the thick line page.
(It's good to make two copies of this because intersecting lines are useful; e.g., for simultaneous equations).

Projecting a grid onto a white board and then drawing on the whiteboard often goes wrong because if you bump the projector it's hard to get it back exactly where it was. Also, if there's any distortion of the image, a ruled line on the board won't appear straight against the grid. So it's best to put the line somehow onto the projector (e.g., using the "thick line" acetate given or a piece of wire) and slide it around. "What's the equation of this line?", etc.

Put up a line (e.g., $x = 4$) and ask pupils to tell you the co-ordinates of any of the points along that line. "What's the same about all those co-ordinates?" The first (x) number is always 4, so we call the line $x = 4$.
You can use a similar process on more complicated equations; e.g., $y = 2x + 3$.

Always start by picking out co-ordinates of several points on the line and looking for some connection between the x-numbers and the y-numbers. Later we can notice that in $y = mx + c$ the gradient is m and the intercept on the y-axis is $(0, c)$.

Tell me the equation of a line that would go through (2,7), etc.

e.g., $y = x + 5$, $y = 3x + 1$, $x = 2$, etc.

1.23.3 How can two lines have the same steepness but not be parallel?

Answer: their gradients are of equal size but opposite sign – one is minus *the other.*
They're reflections of each other in the vertical axis; e.g., gradients 3 and –3.

1.23.4 It's sensible to spend some time on gradient before using it with $y = mx$ and later with $y = mx + c$.

From one point on a straight line to another point on the same line,

Gradient doesn't have to be on co-ordinate axes.
e.g., "Show me with your arm a gradient of 2."
"I'm driving along a road with gradient $\frac{1}{3}$ and then it changes to $-\frac{3}{4}$. Describe what's happened."
Use "thick line" acetate and squared acetate (without axes) to do quick questions: "What's the gradient now?"

$$\textbf{gradient} = \frac{\textbf{distance up}}{\textbf{distance to the right}}$$

Going uphill to downhill, but the downhill is steeper than the uphill was.
Positive gradient means uphill (from left-to-right). Negative means downhill.

Gradient number-line:

–1.0	–0.5	0	0.5	1.0
downhill, getting ← steeper		flat	uphill, getting steeper →	

A gradient of 1 (or –1) is 45° to the axes.

If the gradient is the same all the way along a line it will be a straight line (that's what makes it straight).
You can use the formula $m=\frac{\Delta y}{\Delta x}=\frac{y_B-y_A}{x_B-x_A}$ *for the gradient of the straight line joining* (x_A, y_A) *to* (x_B, y_B)*. Pupils can think about why it works for negative co-ordinates as well as positive.*

1.23.5 People maths. Seat pupils in rectangular array with roughly equal distances separating pupils along the rows and down the columns.
Set the pupil in the far back right corner (from the teacher's point-of-view) as the position (0,0) and then work along the rows and columns assigning each pupil a pair of co-ordinates:

Depending on your classroom layout, you may not be able to do this perfectly. Science labs with benches fixed to the floor are particularly awkward: you could decide to take pupils into the hall or outside.

(4,0)	(3,0)	(2,0)	(1,0)	(0,0)
(4,1)	(3,1)	(2,1)	(1,1)	(0,1)
(4,2)	(3,2)	(2,2)	(1,2)	(0,2)
(4,3)	(3,3)	(2,3)	(1,3)	(0,3)
(4,4)	(3,4)	(2,4)	(1,4)	(0,4)
(4,5)	(3,5)	(2,5)	(1,5)	(0,5)

TEACHER

It may come out different from this depending on your classroom and class size.

This way the xy *grid is the right way round for the class, and the teacher just has to keep alert!*

Go along rows and columns with each person reading out his/her co-ordinates.
"What's the same all the way along that row/column?" e.g., x values all 2.
"Stand up if your y value is one more than your x value." "Stand up if your x value is less than your y value" etc.
"Who hasn't stood up yet?" "What equation could we say that would involve you?"

These instructions give the graphs $x = 2$, $y = x + 1$ *and* $x < y$ *(a region this time, rather than just a line).*

1.23.6 You could investigate the relationship between the gradients of perpendicular lines by drawing, calculating, recording and looking for a pattern. (You don't really need protractors if you use squared paper, or you can use the corner of a piece of paper to check for a right angle.)

Answer: perpendicular gradients multiply to make –1.

So if a line with gradient m_1 *is perpendicular to a line with gradient* m_2*, then* $m_1 m_2 = -1$.

Is it true that any two lines are either parallel or they intersect each other?

Yes in 2 dimensions, but in 3 dimensions it isn't because "skew lines" are non-parallel non-intersecting lines.

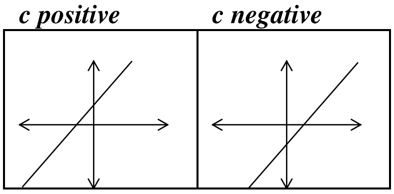
1.23.7 Use a computer graph-plotting program to draw lots of straight-line graphs and look at the effects of changing m or c (big/small, positive/negative).

Pupils could record their results visually in a table (see right).

e.g., investigating positive/negative effect:

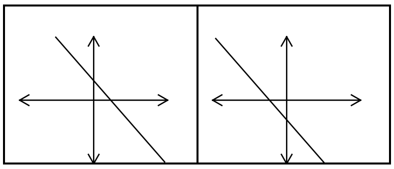
c positive ***c negative***

m positive



In each case, the origin is at the point where the axes cross.

m negative



1.23.8 Open-Ended Questions:

1 Tell me the equations of 4 lines that go through (2,1).

2 Tell me the equations of 4 lines that make a square.

3 Tell me the equations of 4 lines that make a "tilted" square (one where the sides aren't horizontal and vertical).

4 Which of these points is the odd one out because it doesn't lie on the same straight line as the other three?

a (4, 11); b (–2, –5); c (0, –1); d (5, 14)

5 What's special about the equations of parallel lines? What about the equations of a pair of perpendicular lines?

Answers:

e.g., $y = \frac{1}{2}x$; $y = x - 1$; $y = 1$; $x = 2$

lots of possibilities

b; all the others lie on the line $y = 3x - 1$

parallel lines have the same m *value; perpendicular have* m *values that multiply to make –1 (see section 1.23.6).*

1.23.9 Battleships (play in pairs).

You need to use a fairly large grid to make it interesting; e.g., –10 to 10 for both x and y .

Each player picks 4 pairs of co-ordinates for his/her battleships, and they take it in turns to call out equations of straight lines (torpedoes) *which have to come from one of their own ships*. They draw the line and if it goes through a battleship it sinks it (or 2 hits to sink – teacher's discretion!). The player whose ships are all sunk first is the loser.

Pupils could keep a record of torpedo equations in case of disagreement afterwards!

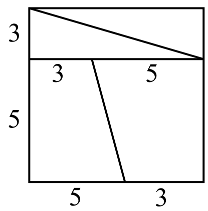
One strategy is to use lines which go through the maximum number of points in the –10 to 10 range; e.g., $x + y = 20$ *would be a poor choice, because it goes through (10,10) only.*

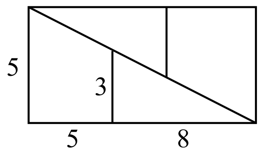
Also, where the paths of the enemy's torpedoes "cross" may be the location of one of his/her ships, but because he/she has 4 there is no guarantee!

1.23.10 Dissection Puzzle.

Use 1 cm × 1 cm squared paper to draw an 8 cm × 8 cm square (area 64 cm^2).

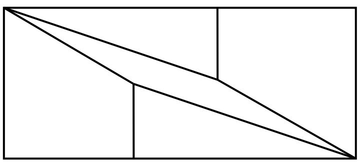
Cut it up as shown below and rearrange it to make a 5 cm × 13 cm rectangle (area 65 cm^2). Where has the extra 1 cm^2 come from?

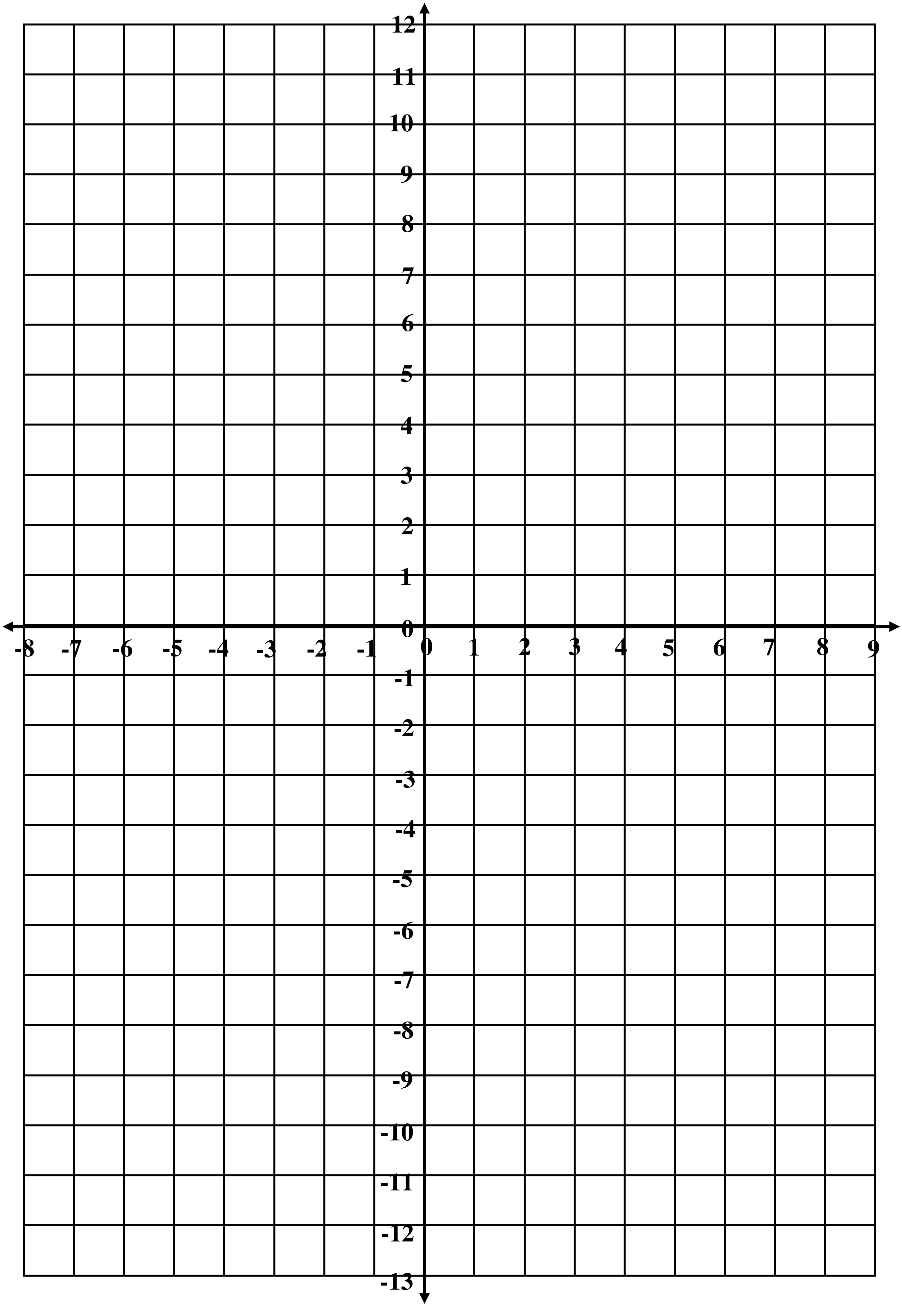




Answer: In the second arrangement, the gradient of the top part of the "diagonal line" is $-\frac{2}{5}$ *, but the bottom part has gradient* $-\frac{3}{8}$ *, so the "diagonal line" is in fact not a single straight line but two lines.*

The exaggerated drawing below shows that the missing 1 cm^2 is contained in the parallelogram gap left in the middle of the shape.

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